Analysis of Geophysical Networks Derived From Multiscale Digital Elevation Models: A Morphological Approach

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***Abstract*—We provide a simple and elegant framework based on morphological transformations to generate multiscale digital elevation models (DEMs) and to extract topologically significant multiscale geophysical networks. These terrain features at multiple scales are collectively useful in deriving scaling laws, which exhibit several significant terrain characteristics. We present results derived from a part of Cameron Highlands DEM.**

***Index Terms*—Channel and ridge networks, digital elevation model (DEM), fractal dimension, morphology, multiscaling.**

I. Introduction

PROCESSING of remotely sensed information in both spatial and frequency domains has received notable attention. Various types of remotely sensed terrestrial, lunar, and planetary surface data are analyzed in order to understand several characteristics of geoscientific interest in spatio-temporal mode. These interests lie primarily in the fields of hydrology, geomorphology, geology, geophysics, ecology, and environment. The application of remote sensing in these fields is realized with very promising results. One of the data derivable from remote sensing data is a digital elevation model (DEM) from which a lot of information can be extracted for the purposes of flood mapping, resource management, telecommunications mapping, military mapping, city planning, and so on. Thematic information extraction from DEM images with the application of spatial analysis and sensing techniques provide scientists with invaluable information in terrain characterization. With the advent of interferometry techniques, it is now possible to precisely generate DEMs with fine resolution and high accuracy [1], [2].

Regions with varied degrees of concavity and convexity represent various degrees of terrain complexities. These complexities explain various physiographic and geomorphic processes. The abstract structures of concave and convex zones represent the valley and ridge connectivity networks, respectively. These two unique topological networks have immense use in characterizing the surficial terrain quantitatively via morphometry [3], [4], hypsometry [4], allometric scaling [5]–[7], and granulometry [8]. Valley network extractions from DEMs have been done by O'Callghan and Mark [9] and Tarboton *et al.* [10]. Valley connectivity networks are available in surveyed topographic maps. Nevertheless, no topographic map provides ridge connectivity networks, and its extraction needs to be emphasized in addition to the extraction of valley networks.

The main objective of this letter is to explore another method based on morphological transformations to analyze the network where additional properties would be provided for terrain characterization. We employed the morphological transformations to extract the networks and fractal techniques to characterize the DEM. There are four significant steps of equal importance which include: 1) DEM derivation from remotely sensed data; 2) generation of DEMs at multiple resolutions; 3) extraction of topological information from DEMs; and 4) employing the topological information in terrain modeling and characterization studies. The latter three steps are addressed in this letter.

The Malaysian government, under the coordination of the Malaysia Center for Remote Sensing (MACRES), participated in the Airborne Synthetic Aperture Radar/Topographic Synthetic Aperture Radar (AIRSAR/TOPSAR) PACRIM program jointly organized by the National Aeronautics and Space Administration and the Commonwealth Scientific and Industrial Research Organization. Polarimetric AIRSAR and interferometric TOPSAR data are used for terrain-related analysis. In this letter, we analyze the interferometrically derived TOPSAR DEM of Cameron Highlands region of Malaysia [Fig. 1(a)] situated between $101°17'$ and $101°19'$ E longitude and $4°31'$ and $4°34'$ N latitude. This region comprises a series of mountain stations at altitudes between 500–1300 m.

The organization of this letter is as follows. Section II gives basic definitions of morphological operators. The methodology to generate multiscale DEMs and to extract channel and ridge networks from the multiscale DEMs is presented in Section III. The estimation of fractal dimensions of both channel and ridge connectivity networks that characterize terrain is presented in Section IV. Finally, concluding remarks are given in Section V.

II. Basic Definitions and Notations

We utilize set-theory-based mathematical morphology transformations [11] that are popular in object recognition and representation studies. We examine the geometrical and topological structures of a DEM by matching it with structuring elements at various locations in the DEM. Variation of characteristic information such as size, shape, origin, and orientation of the structuring element facilitates further investigation in understanding several interrelationships. Four morphological

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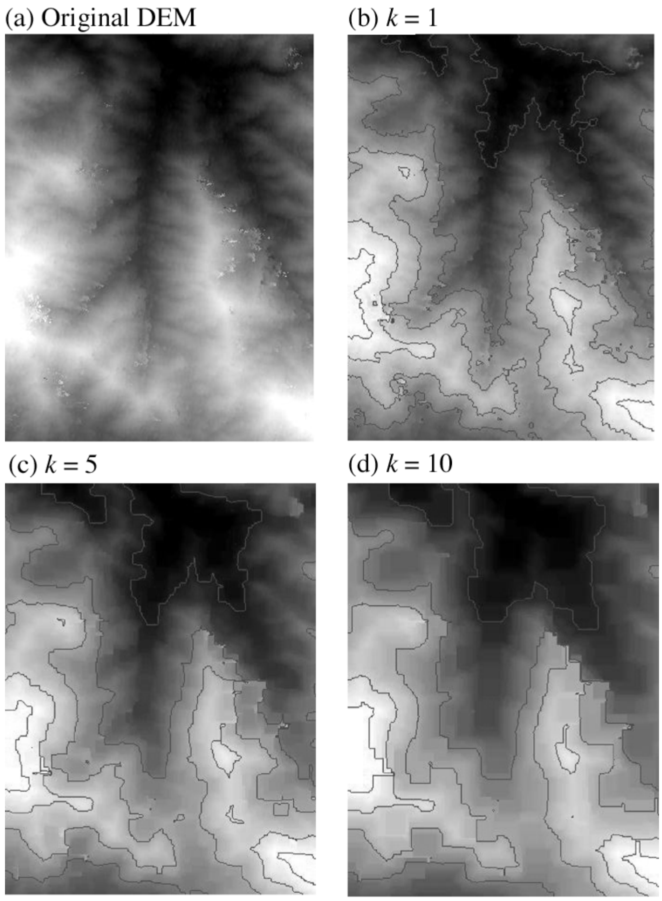

Fig. 1. (a) DEM of Cameron Highlands. (b)–(d) DEM at multiple scales.

operators are employed. Dilation and erosion operations are performed for expansion and contractions, respectively, while opening and closing operations are performed to smoothen the elevation contours that are conspicuous in the DEM. The DEM as a grayscale function $f(x, y)$ is seen as a finite subset of two-dimensional discrete space $\mathbf{Z}^2$. Each pixel represents an elevation region distributed spatially. Brighter and darker gray levels in a DEM represent higher and lower category elevations, respectively.

We define the grayscale dilation (erosion) of a DEM $f(x)$ at a given pixel x as the maximum (minimum) value of the image in the window defined by the structuring element B when its origin is at x. This is mathematically expressed as

$$(f \oplus B)(x) = \max\left[f(x+b)\right], \qquad b \in B \tag{1}$$

$$(f \ominus B)(x) = \min\left[f(x+b)\right], \qquad b \in B. \tag{2}$$

Erosion is the dual of dilation as eroding foreground pixels is equivalent to dilating the background pixels. Opening of DEM f by B is achieved by eroding f and followed by dilating with respect to B and is mathematically shown as

$$(f \circ B) = [(f \ominus B) \oplus B]. \tag{3}$$

Closing of f by B is defined as the dilation of f by B followed by erosion with respect to B, which is mathematically represented as

$$(f \bullet B) = [(f \oplus B) \ominus B]. \tag{4}$$

Opening eliminates specific image details smaller than B, removes noise, and smoothens the boundaries from the inside, whereas closing fills holes in objects, connects close objects or small breaks, and smoothens the boundaries from the outside. Multiscale opening of scale n is defined as erosion of the image by B for n-times followed by dilation with the same B for n-times. By duality, multiscale closing of scale n is defined as dilation of f by B for n-times followed with erosion by B for n-times. These multiscale opening and closing transformations are mathematically represented as $(f \circ B_k) = [(f \ominus B_k) \oplus B_k]$ and $(f \bullet B_k) = [(f \oplus B_k) \ominus B_k]$, respectively, where the scaling factor $k = 1, 2, 3, \ldots, K$.

III. Methodology

A. Multiscale DEM

Important problems like feature detection and characterization often require analyzing DEMs at multiple spatial resolutions. In the recent past, multiscale DEMs were generated via smoothening by the application of diffusion equation with Green's function (Gaussian function) [12]. As the resolution decreases, the shape of the joint density changes, and the edges cannot be preserved. This approach of multiscale DEMs generation blurs or shifts contours in DEMs. Recently, nonlinear filters have been used to obtain images at multiresolution due to their robustness in preserving the fine details. Nonlinear multiscale transforms include morphological transform [11], [13]–[16], partial differential equations [17], and extended nonlinear wavelet transforms [18]. In this letter, we adopt nonlinear morphological opening and closing.

Multiscale morphology is defined as morphological operations with scalable structuring elements. We generate multiscale DEMs of a part of Cameron Highlands by employing multiscale morphological openings and closings with structuring elements (square in shape) of successively scaled spatial dimensions. The size of the structuring element is incremented with the scaling parameter k from 1 to 10 (for $k = 1$, size of B is 3×3, for $k = 2$, size of B is 5×5, and so on). Fig. 1(a) represents a DEM of the study area where its pixel values are in the range of 0–255. The DEMs f at multiple scales that are generated based on closing are shown in Fig. 1(b)–(d).

B. Channel and Ridge Connectivity Networks

The two topologically significant networks are channel and ridge networks, which are the abstract structures of concave and convex zones of the DEMs, respectively. The paths of these extracted networks are the crenulations in the elevation contours. Precise isolation of all possible crenulations from DEMs is possible with systematic use of nonlinear morphological transformations. By using a part of Cameron highlands DEM at decreasing resolutions, ridge and channel networks are extracted. The DEM f is first eroded by structuring element, B_n with $n = 1, 2, \ldots, N$, and the eroded DEM is opened by

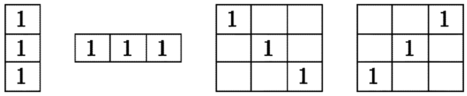

Fig. 2. Line segment structure elements in four directions.

B of the smallest size. The opened version of each eroded image is subtracted from the corresponding eroded image to produce the nth level subsets of the ridge network. Union of these subsets of level $n = 0$ to N gives the ridge network for the DEM [19], [20]. For channel networks, duality of the morphology approach is used where the DEM f is first dilated by structuring element B_n and the dilated DEM is closed by B (the smallest structuring element). The closed version of each dilated image is subtracted from the corresponding dilated image to produce the nth level subsets of the channel network. Union of these subsets of level $n = 0$ to N gives the channel network for the DEM. In this investigation, two sets of network are extracted. Besides following the work on [20] using square template, we extend our work by employing line segment structuring elements as shown in Fig. 2 for B_1. B_n is the increasing version of B_1 for $n = 1, 2, 3, \ldots, N$. Line segments in four different orientations (Fig. 2) are used as their shapes are closer to the linear network of ridge and channel. The mathematical expressions employed to extract both ridge $\mathrm{RID}(f)$ and channel $\mathrm{CH}(f)$ networks are as follows:

$$\mathrm{RID}_n^i(f) = \left[\left(f \ominus B_n^i\right) \setminus \left\{\left[\left(f \ominus B_n^i\right) \ominus B_1^i\right] \oplus B_1^i\right\}\right] \quad (5)$$

$$\mathrm{RID}(f) = \bigcup_{\substack{n=0\\i=1}}^{\substack{4\\N}} \left[\mathrm{RID}_n^i(f)\right] \quad (6)$$

$$\mathrm{CH}_n^i(f) = \left[\left(f \oplus B_n^i\right) \setminus \left\{\left[\left(f \oplus B_n^i\right) \oplus B_1^i\right] \ominus B_1^i\right\}\right] \quad (7)$$

$$\mathrm{CH}(f) = \bigcup_{\substack{n=0\\i=1}}^{\substack{4\\N}} \left[\mathrm{CH}_n^i(f)\right]. \quad (8)$$

The $\setminus$ symbol in (5) and (7) denotes subtraction. The $\bigcup$ symbol in (6) and (8) denotes the union of all level subsets, and it is implemented with logical OR operation. Note that the opening and closing in (5) and (7), respectively, are performed with the smallest structuring elements of the concerned direction, i. Thus, the four basic directions (Fig. 2) are considered and not for the exotic directions of larger sized squares. By using thresholding process, the extracted networks are converted into binary form, where all nonzero pixels are assigned as 1 and zero pixels remain its value. A morphological thinning approach is used to thin the network by reducing all lines to one-pixel-wide thickness, and the thinned lines are the skeleton of the thresholded network image. All these skeletal pixels are considered to be the medial axes, and they represent the networks of the DEM in this case study. By using line segments in four orientations as structuring elements (Fig. 2), two unique topological connectivity networks [Fig. 3(a)–(c) and (d)–(f)] are derived from multiscaled DEMs.

Channel networks extracted from multiscale DEMs are compared with that of surveyed topographic map, and the results are found to be close to the networks in the map. A sparse network would indicate that its terrain is relatively simplistic, while an intricate network would indicate a terrain with higher complexity. The network complexity can be better quantified via fractal dimensions which are potential indicators to explore more links with geomorphic processes. This letter provides further insights in the role of network complexity in understanding the landscape evolution.

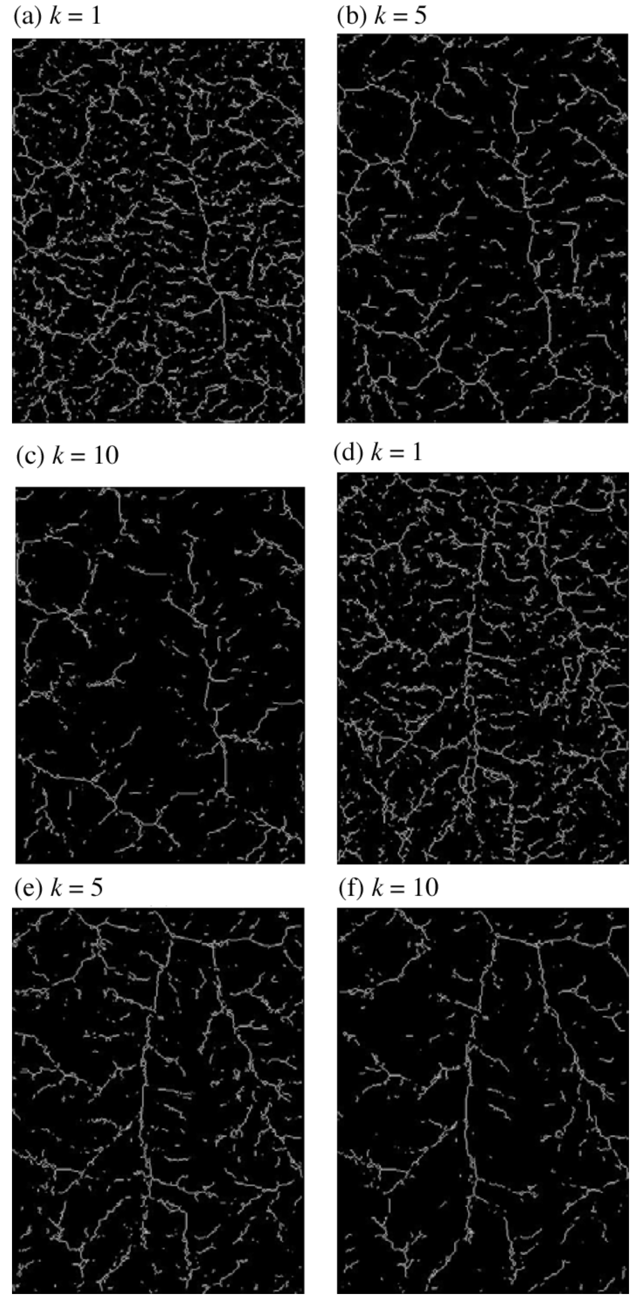

Fig. 3. (a)–(c) Multiscale ridge networks and (d)–(f) multiscale channel networks extracted from corresponding multiscale DEMs shown in Fig. 1(b)–(d).

IV. Fractal Dimension of Networks

Wider application of morphology and scaling based analyses and characterization is addressed in this section. Spatial resolutions of DEM data limit our ability to study terrain characterization. Hence, to describe the state of the landscape,

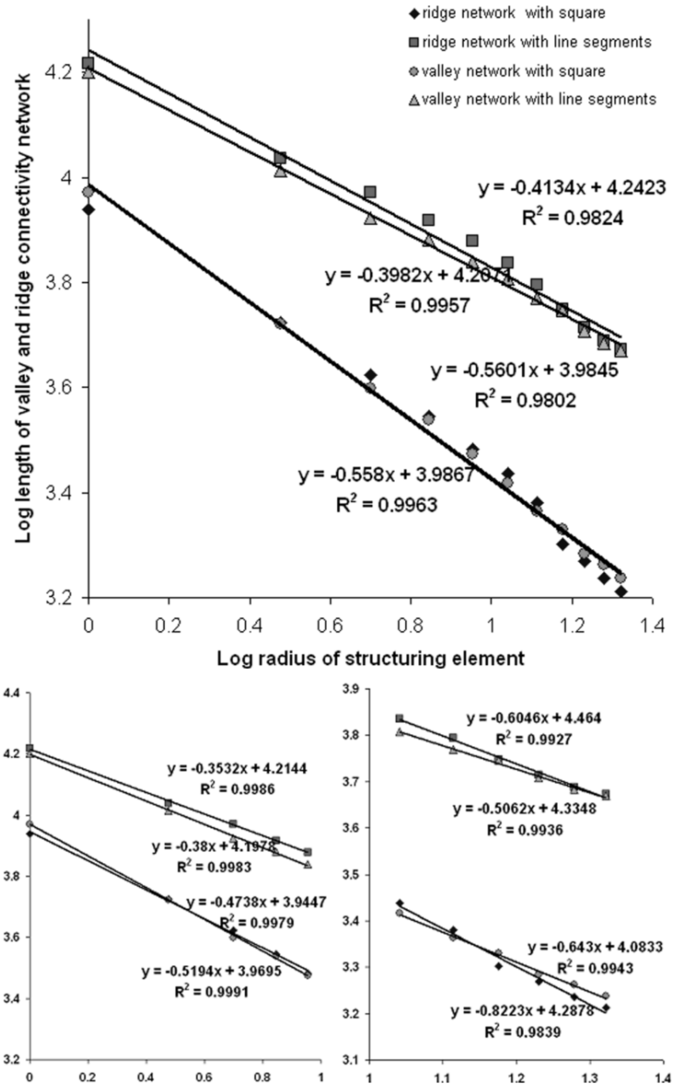

Fig. 4. Graphical relationships between radius of structuring element and length of multiscale ridge and valley networks.

resolution-dependent parameters derivable from DEM are of limited use. A more reliable approach is the application of resolution-independent parameters, which describes the rate of change in the lengths of geophysical networks across the resolutions. In order to achieve this goal, we employ multiscale nonlinear morphological transformations to generate DEMs at multiple resolutions. The networks derived from these multiscale DEMs are employed to derive a simple resolution-independent power-law-like dimension. Analysis of these networks of Cameron Highlands through scaling is shown.

Fig. 4 depicts the relationship between the degree of multiscaling (in terms of the radius of the structure element) used to generate multiscale DEMs, and the lengths of the ridge and valley networks (in terms of the number of pixels of the single line thickness networks in both horizontal and vertical directions and $\sqrt{2}$ times of number of pixels of network in diagonal directions) extracted from respective multiscale DEMs. The axes on the graph represent logarithmic values of radius of structuring element and lengths of valley and ridge connectivity networks. This relationship yields a power-law form of $l \sim r^{\alpha}$, where l and r denote length of network at different scale and radius of structuring element, respectively. α, as the scaling exponent, is the fractal dimension of the network. This relationship depicts that similar trends have been followed for both ridge and valley connectivity networks and describes the scaling properties of the terrain where the density of the networks decreases as the resolution decreases. This change is due to the fact that the diffuse character of the DEM increases as the size of the structuring template increases. This relation can be reversed and estimation of lengths of these networks can be made from coarse-scale information.

The lengths of ridge and valley networks extracted by employing line segment structuring elements are significantly more than that of the networks extracted by convex type of square template. We plot these lengths as functions of radius of structuring element. The gradients of best fit lines of these plots indicate that the rate of change in the lengths of the networks, across multiple resolutions. The rate derived by segment-like structuring elements is slower than that of the networks derived by square elements.

For the present region, the power-law exponents derived (Fig. 4) are 0.398 and 0.413, respectively, for valley and ridge connectivity networks extracted using line segments as structuring elements. By using square as a structure element, these values are 0.558 and 0.560. Besides that, we also split the plot into two different scales: the x axis depicts the radius of structuring element as low scale ($k = 0$ to 4) and high scale ($k = 5$ to 10). The splitting is done based on the plot shown on the global scale. The power exponents derived from these new plots show that the rate of changes is smaller in the low scale as compared to the high scale. It is observed that the textured variation (low scale) is less than the structured variation (high scale).

The complexities and intricacies of valley and ridge network change with various types of topography; therefore, network length is considered as an important parameter for complex geometry of valley and ridge. Generally, hilly terrain possesses a higher value of exponent as compared to nonhilly terrain. The reason is that the rate of change in elevation of hilly terrain across resolution is higher than nonhilly terrain. Relatively, the network intricacies will change more rapidly for hilly terrain. Since the power-law exponent is sensitive to structure elements of different shape (shape-dependent), its value would be related to the shape and roughness of the terrain. Thus, these power laws can be related to terrain roughness characteristics. Further analyses of these two networks provide the morphologies of convex zones and hillslopes.

The directions of channel networks in fluvial and tidal systems are, respectively, determined by topography and local hydrodynamics [21]. The morphology-based framework shown here is useful to extract the networks from both fluvial and tidal systems. Scale invariance tendencies are so prevalent in fluvial systems, whereas it is reported [21] that analyses reveal complete lack of such tendencies in tidal channel systems. In order to report basic differences between tidal and fluvial network forms, this length criterion and fractal dimensions of networks derived from fluvial and tidal systems will be useful. In this letter, fractal measures are shown as sample for the ridge and valley networks extracted from a fluvial system. Such a study provides a basis for comparing relationships in measures of tidal and fluvial landforms. It can also be used as a powerful criterion for the classification among subbasins in a large basin. Basins with different topography structure have different network geometry and densities. Emerging river network patterns can be considered to relate with basic underlying physical mechanisms involved in the formation of landscapes of varied complexities. The differences will be reflected by the fractal dimension as this exponent is computed based on network densities across multiple scales.

The analyses of networks extracted by structuring elements of each direction provide new insight to understand the direction-specific terrain complexity. The classification based on direction-specific network information can be further used to explore links with surficial processes. These direction-specific ridge and valley networks are abstract structures of convex and concave zones in the specific direction. Characterization of a surface that is perturbed due to tectonic and/or climatic forcing can be done by characterizing corresponding direction-specific networks of both types.

In our follow-up study, we derive watershed-wise fractal-based power-laws, and shape-size complexity measures via grayscale granulometries to draw precise relationships. These studies would be useful for watershed characterization.

V. Conclusion

In this letter, a multiscale-morphology-based method is proposed to: 1) generate multiscale DEMs and 2) extract channel and ridge networks from these multiscale DEMs. We provide a simple scaling law from the relationship shown by considering the lengths of unique networks derived from multiscale DEMs as functions of radius of the structuring element. This scaling exponent is named as fractal dimension which is the slope of the best fit line of $\mathrm{Log}(l) \sim \log r$. The higher the exponent the higher is the rate of change in the ridge and/or valley network density. However, this rate of change across resolutions is slow in the relatively flat terrains. In this way, one can quantify the terrain complexities via these scaling exponents. In-depth results can be derived via morphometry, hypsometry, allometry, and granulometry methods, and this provides firm quantitative comparisons in unraveling the topologically important terrain characteristics.

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