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Morphological description and interrelation between force and structure: a scope to geomorphic evolution process modelling

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Abstract. The concept of force is described in morphological terms. The interrelations among the original morphological constitution, the force influencing a system, and the type of process that the system can undergo, are described with a morphological representation. Five laws of structures are proposed. The reaction of a geomorphic feature to a perturbation created by means of morphological force is discussed. The morphological dynamics of a system subjected to undergo morphological process due to perturbation is qualitatively modelled through graphic analysis. However, the multi-temporal satellite data is the main source of information to record the morphological changes of a geomorphic feature in a time sequential mode and further to model the morphological dynamics. The applications of the concepts that are discussed and described systematically in this paper are foreseen to achieve cogent models that describe the behaviour of morphological changes.

1. Introduction

The morphological behaviour of certain objects depends upon the original morphological constitution, type of the force they are subjected to, and the type of process undergone. During the evolution process, some systems may disintegrate and then disappear. Some morphologically stable systems may become unstable, and *vice versa*. Many possibilities from stable to unstable, and/or chaotic behaviour in the morphology may be encountered during the evolution process of certain geomorphic features. Such geomorphic features can be broadly categorised as the features where the changes can be noticed at short time intervals (e.g., lake), and those in which changes can be noticed at long time intervals. The difference between short and long time periods is yet to be defined. To model the dynamically changing morphology of a geomorphic feature by incorporating the concepts that follow in this paper, various types of remotely-sensed data available in temporal sequence to model the morphological changes are essential. What follows in this section includes the use of multi-temporal satellite data to make an attempt to model the morphological dynamics. To model the morphological changes that have been collated from multi-temporal satellite data the application of mathematical morphological concepts, fractal geometry and chaos theory are foreseen in the modelling and simulation studies of geomorphic evolution process. The study of a specific geomorphic feature undergoing transformation across times enables the evaluation of the type of force and the process undergone. The force responsible for the transformation can be defined and designed in morphological terms. The sequential steps to model the morphological dynamics include:

1. Isolation of the feature from multi-temporal satellite data using image processing techniques. This should be done to find out the morphological changes across time intervals. In this phase of analysis in the modelling, a remote sensing scientist needs to extract a geomorphic object of interest which should essentially be in a closed form. It is quite obvious that a feature generally undergoes either one or any combination of the four possible morphological processes (details of these processes are described in § 3). Depending upon the complexity in the morphological dynamics of the feature, certain conditions have to be imposed on the force to which the feature is subjected.

2. Derivation of morphological process can be done by probing the original feature by comparing it with that of the feature at specific time intervals. Common possibilities of morphological processes that a system can undergo are detailed in § 3 from which laws of structures are proposed. The type of morphological process that the system has undergone can be studied only by considering the feature under investigation at several time intervals. Hence, the advent of remote sensing technique which gives data at sufficiently close intervals, may be realised.

3. Design of morphological force by investigating the feature at different time intervals with pre-defined morphological forces. When the feature is considered at two different time periods a possible morphological force with characteristic information can be defined (detailed procedure to define morphological force is given in § 2).

4. Multi-temporal satellite data is needed to define the morphological force with which the future structure can be constructed from the structure at time t (or) to reconstruct the original structure from the structure at time $t+1$. However, this is perforced by a need to find out an order in the form of intensity of the morphological force that is represented with size being one of the characteristics, besides orientation, direction, and shape (§ 2 gives more details). To find out the order in the morphological force, the feature in temporal domain should be considered. Multi-temporal satellite data fulfil this. This data enables us to determine the rate of change in the structure. The difference in the morphological characteristics in a geomorphic feature from time to time can then be compared and analysed further to define morphological force and process. This helps in modelling the morphological dynamics.

Firstly, certain basic morphological transformations and the concept of structuring element (Matheron 1975, and Serra 1982) are considered as the basis for this study.

1.1. *Mathematical representation of morphological processes*

Certain important transformations from the field of mathematical morphology such as *erosion*, *dilation*, *closing*, and *opening* are considered in this study. Mathematical morphology based on set theory concepts is a unique approach in the analysis of geometric properties of different structures. The main objective is to study the geometric properties of a natural feature represented as a binary image by investigating its microstructures by means of structuring elements, following Serra's concept (1982). The discrete binary image, M, is defined as a finite subset of Euclidean two-dimensional space, R^2. The following are the mathematical representations of different basic morphological transformations.

Translation: Let D be a sub-image of the Euclidean plane, R^2. The translation of the image D by a point, x, in R^2 is denoted by $D+x$ and defined as

$$D+x=\{d+x : d \in D\}. \tag{1}$$

Dilation: Let M and S be sub-images of Euclidean plane, R^2. The dilation of an image, M, with structuring element, S, denoted by $D(M, S)$ is defined as

$$D(M, S) = M \oplus S = U_{s \in S} M + s = \{x : (-S + x) \cap M \neq \emptyset\}. \tag{2}$$

Erosion: Let M and S be sub-images of Euclidean plane, R^2. The erosion of an image, M, with structuring element, S, denoted by $E(M, S)$ is defined as

$$E(M, S) = M \ominus (-S) = \cap_{s \in S} M + S = \{x : S + x \subseteq M\}. \tag{3}$$

where $-S = \{-s : s \in S\}$, i.e., S rotated 180° around the origin. Two consecutive erosions and dilations can be respectively represented as $(M \ominus S) \ominus S = M \ominus S_2$, and $(M \oplus S) \oplus S = M \oplus S_2$. The dilation followed by erosion is called the *closing* transformation, and the reverse process is the *opening* transformation. These two cascade processes are *idempotent*. The multi-scale approach of these cascade processes (Maragos and Schafer 1986) are not idempotent.

This paper is organised as follows. In §2, the concept of morphological force is described. Using this concept the properties of structures (structure is used to denote 'the expression of external morphology of the objects' such as geomorphic features) subjected to morphological forces and the various processes involved are described in §3. Also, a sample study has been carried out for a better understanding of the morphological dynamics. Based on these processes, five laws of structures have been proposed in this section. A schematic representation of the procedures that can be adopted to model the geomorphic evolution process is presented in §4. In §5, a model to study morphological dynamics is presented qualitatively. In this section, a deterministic approach has been followed to model the dynamics of transcendentally generated fractal lake under specified morphological transformation.

2. Morphological force

Both homogeneous and nonhomogeneous effects are likely to operate simultaneously in the evolution of a geomorphic system. The recognition of such concurrent characters in geomorphic evolution is significant due to endogenic forces, i.e., tectonic character (systematic) and exogenic forces (nonsystematic). It is assumed that the geomorphic evolution process, in general, is based on such forces acting concurrently. The impact of such concurrent forces in a geomorphic system can be studied by taking into account the morphological changes that have occurred in temporal sequence. It is also assumed that the degree of deformation depends upon the intensity of collective or concurrent forces. Hence, the deformed portion of the geomorphic feature is taken as the basis to study the geomorphic dynamics systematically. The concept of morphological force is defined in order to study the dynamics of geomorphic evolution process.

2.1. *Definition of morphological force*

The morphological force which is referred to as structuring element in mathematical morphology is defined here as an entity due to which deformation occurs in a geomorphic system. This entity considered in terms of shape is a collective form of exogenic and endogenic forces. The universe of all possible morphological forces in the form of shapes is vast. In reality, a geomorphic structure will have an objective meaning when it interacts with morphological force. This point makes the study of dynamics of geomorphic evolution process most interesting. Such a study needs proper identification of characteristics of morphological force (detailed in figure (1)) which interacts with the system. The structural aspects of morphological force and the type of interaction with system can be obtained at a temporal sequence. It is intended to study the impact of this force on a specific structure. This concept of force is thoroughly used to create perturbation. Since every structure is prone to

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Force shape as triangle

(*a*)

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangle force shape of size 1, $S_{tri,\,1}$

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangle force shape of size 1, $S_{tri,\,1}$

$$\begin{matrix} 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangle force shape of size 2, $S_{tri,\,2}$

(*b*)

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&*&0&1&0&0\\ 0&1&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangular force of size 1 with origin at (4, 2), $S_{tri,\,1,\,((4,,\,2)}$

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&0&0&1&0&0\\ 0&1&*&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangular force of size 1 with origin at (4, 2), $S_{tri,\,1,\,((3,,\,3)}$

(*c*)

$$\begin{matrix} 0&0&0&0&1&0&0&0&0\\ 0&0&0&1&0&1&0&0&0\\ 0&0&1&0&*&0&1&0&0\\ 0&1&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangular force shape of size 1 with origin at (4, 2), with upward direction, $S_{tri,\,1,\,((4,,\,2)}\uparrow$

$$\begin{matrix} 1&1&1&1&1&1&1&1&1\\ 0&1&0&0&0&0&0&1&0\\ 0&0&1&0&*&0&1&0&0\\ 0&0&0&1&0&1&0&0&0\\ 1&1&1&1&1&1&1&1&1 \end{matrix}$$

Triangular force shape of size 1 with origin at (4, 2), with downward direction, $S_{tri,\,1,\,((3,,\,3)}\downarrow$

(*d*)

Figure 1. Characteristics of force (*a*) shape, (*b*) size, (*c*) origin, and (*d*) orientation.

changes due to perturbation caused by a force across discrete time intervals, both structure and force need to be defined in terms of shapes. Hence, a theory of morphological force in the form of a shape that will associate in a clear way, the geometric characteristics of force with morphological significance of the transformation needs to be developed.

2.2. *Characteristics of morphological force*

To model the structural dynamics, force is represented in the form of shape which also possesses other characteristics like size, origin, and orientation (figure 1). The change in any of the characteristics of force shows impact on the structure during transformations. It is envisaged that force can be properly defined and designed in the form of shape considering the changes that occur in the morphology of the structure across time periods, as detailed in §2.3. The impact of each of the characteristics of force is briefly described as follows.

Shape: In morphology, the term force broadly categorised as homogeneous and nonhomogeneous is visualised as symmetric and asymmetric respectively (figure 2). If forces to expand and shrink the ideal system are homogeneous with similarity in characteristics, there will be no variation between original and transformed structures. In contrast, an irregular structure may become regular under the process of expansion followed by contraction, continuous expansion or contraction followed by expansion, provided the force is of a homogeneous type. It is also true that a regular structure becomes irregular when the force is of a heterogeneous type.

Size: During the process of cascade of expansion-contraction detailed in §3.2, if the size of the force is large, its intensity is relatively high compared to the force of smaller size. The forces of larger size will transform the original structure in such a way that the structure resembles the shape of the force. Also, in the case of a smaller force incessantly expanding the structure, the cumulative force acting on the structure becomes large after a certain time period and transforms the structure such that it resembles the shape of the force. In such a case the size of the force may be represented as S_1 in first iteration, S_2 in the second iteration and so on up to S_n in nth iteration, where $S_1 < S_2 < S_3 < \ldots < S_{n-1} < S_n$. The structure will reach a state of convergence (the state after which all the succeeding transformed structures are morphologically similar), if the size of the force, say S_n, is larger than the size of the original structure. Under the process of cascade of expansion-contraction by means of a force of size S_n, the structure, M, reaches to a state of convergence provided $S_{n-1} < M < S_n$.

Origin: In the structural transformation, several possibilities, viz., contraction, expansion, contraction followed by expansion and expansion followed by contraction can be studied. In all these processes, besides all characteristics, the origin also shows

$$\begin{matrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{matrix}$$

(*a*)

$$\begin{matrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{matrix}$$

(*b*)

Figure 2. Type of forces (*a*) homogeneous, (*b*) nonhomogeneous.

an impact on the structure in terms of deformation. The impact of the morphological force of the same shape, with a different origin has been shown diagrammatically (figure 3).

Direction: The direction of the force to expand the structure may be different from the direction of the force to contract. An asymmetric force can have many possible directions.

2.3. *Design of force as shape*

To design the morphological force, it is essential to have the structure at two time periods and set of predefined forces with all possible characteristics. For a better understanding, a hypothetical geomorphic feature at two discrete time periods (figures 5 (*a*, *b*)) is considered. The difference image (figure 5 (*c*)) of the geomorphic feature between these two discrete time periods needs to be investigated with a set of symmetric morphological forces (where $S = \hat{S}$). A morphological force which is contained in the difference image is considered as the force responsible for the

$$\begin{array}{cccc} 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1 \end{array} \qquad \begin{array}{cccccc} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1 \end{array} \qquad \begin{array}{cccc} 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1 \end{array} \qquad \begin{array}{cccc} 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1 \end{array}$$

Original Image $\quad$ $M \oplus S1$ $\quad$ $M \oplus S2$ $\quad$ $M \oplus S3$

$$\begin{array}{cccc} &1&1&1\\ \text{Origin}\rightarrow&1&1&1\\ &1&1&1 \end{array} \qquad \begin{array}{ccc} 1&1&1\\ 1&1&1\\ 1&1&1\\ &\uparrow&\\ &\text{Origin}& \end{array} \qquad \begin{array}{ccc} &&\text{Origin}\\ &&\downarrow\\ 1&1&1\\ 1&1&1\\ 1&1&1 \end{array}$$

S1 $\quad$ S2 $\quad$ S3

Figure 3. Shows expansion process with a square type of morphological force on the structure with different origins.

$$\begin{array}{ccc} 0&1&0\\ 1&1&1\\ 0&1&0 \end{array} \qquad \begin{array}{ccccc} 0&1&1&1&0\\ 1&1&1&1&1\\ 1&1&1&1&1\\ 1&1&1&1&1\\ 0&1&1&1&0 \end{array} \qquad \begin{array}{ccccccc} 0&0&1&1&1&0&0\\ 0&1&1&1&1&1&0\\ 1&1&1&1&1&1&1\\ 0&1&1&1&1&1&0\\ 0&0&1&1&1&0&0 \end{array}$$

Rhombus $\quad$ Octagon $\quad$ Hexagon

(*a*) $\quad$ (*b*) $\quad$ (*c*)

Figure 4. Various types of morphological forces (*a*) rhombus, (*b*) octagon, and (*c*) hexagon.

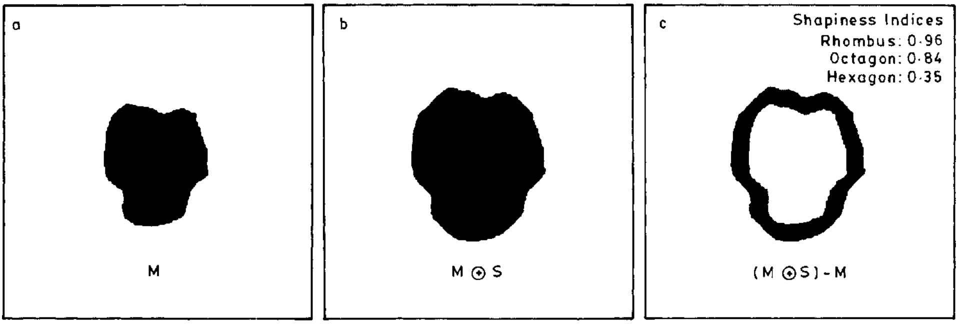

Figure 5. (*a*) Hypothetical geomorphic feature at time t, (*b*) geomorphic feature at time $t+1$, and (*c*) difference in geomorphic feature from time t to $t+1$.

deformation of the structure. All the characteristic information of such a force should be known. For the sake of simplicity, only pre-defined symmetric forces such as rhombus, octagon, and hexagon (figure 4) are considered. In order to find out the force responsible for the deformation, the pattern spectrum procedure detailed by Maragos (1989) is used. The difference image should be allowed to undergo a cascade of contraction-expansion process by means of all pre-defined forces. Through this investigation shapiness indices relative to rhombus, octagon and hexagon are computed as 0·96, 0·84, and 0·35 respectively. From these shapiness indices it is inferred that the force contained in the difference image is rhombus.

Such an investigation of the difference image with asymmetric forces is often required while considering natural geomorphic features at a temporal domain in order to design the appropriate morphological force.

The procedure detailed in this section emphasises the procedure to find out a tool in terms of shape that influences a geomorphic structure. Across time intervals, certain geomorphic features undergo several transformations. In turn, the morphological constitution will change as time progresses. In general, these morphological changes are due to two broadly categorised physical forces namely, endogenic and exogenic. However, a clarification has been given as to how a new entity called morphological force can be defined by considering a feature of interest across time intervals. To define such a morphological force that encompasses both endogenic and exogenic forces one needs to know the morphology of the feature in temporal mode. This feature at two different time periods will be considered to find out a morphological force. To fulfil this, the remote sensing data is of immense use. Nevertheless, the separation of endogenic and exogenic forces from morphological force still needs in depth investigation. However, the morphological force can be decomposed into endogenic and exogenic forces (which are also in the form of shapes) by making use of resolution property. But, this is not within the scope of the present investigation.

3. Morphological processes

In this section, various processes induced by a force are described. The cumulative morphological force can be mathematically represented as

$$S \oplus S \oplus S \oplus S \oplus \ldots \oplus S = S_n. \quad (4)$$

Its diagrammatic representation is shown in figure 1 (*b*).

3.1. *The process of continuous expansion*

Continuous expansion of the structure by means of a specific process is represented as follows

$$\{[(M \oplus S) \oplus] \oplus S\} \ldots \oplus S = (M \oplus S_n) \neq \varnothing \tag{5}$$

Under the process of continuous expansion due to homogeneous force, an irregular structure may become regular. During this process, the structure reaches the state of convergence if it is contained in the force.

$$\left.\begin{array}{ll} (M \oplus S_{(n-1)}) & \text{where } M \subseteq S_{n-1} \\ [M \oplus S_{n-1}] \equiv M \oplus S_n \neq \varnothing & \end{array}\right\} \tag{6}$$

$\equiv$ is a symbol used to denote geometrical similarity.

A structure remains morphologically invariant during this process if the force is geometrically similar to the original structure.

3.2. *The process of cascade of expansion followed by contraction (C-EC)*

The process of cascade of expansion-contraction (C-EC) may be represented as

$$\{[(M \oplus S) \ominus S] \oplus S\} \ominus S = (M \oplus S) \ominus S \tag{7}$$

Based on the multi-scale representation of morphological processes (Maragos and Schafer 1986), the effect of cumulative force during the process of C-EC can be represented as

$$\{[(M \oplus S) \oplus S] \oplus \ldots \oplus S\} \ominus S] \ominus S] \ominus \ldots \ominus S = (M \oplus S_n) \ominus S_n \neq (M \oplus S) \ominus S \tag{8}$$

The characteristics of the force are not represented since they are assumed to be identical from one cycle to another. It will be interesting to study this process where expansion is followed by contraction with geometrically dissimilar forces. During this process, only an ideal structure reaches to its original state when it is subjected to a force with identical characteristics for both expansion and contraction. The process of C-EC will show its impact on the structure till the structure attains the state of convergence. Thus, if a structure reaches the state of convergence after $(n-1)$th cycle, there will be no morphological change in the transformed structure from $(n-1)$th cycle to (n)th cycle. This can be shown as

$$(M \oplus S_n) \ominus S_n = [M \oplus S_{(n-1)}] \ominus S_{(n-1)} \tag{9}$$

For instance, if an elliptical structure shown in figure 6(a) is transformed under the process of C-EC by a force of size S_n, the structure will reach to the state of convergence after one iteration. A change in any of the characteristics of force during expansion or contraction will have its impact on the transformed structure. The force to expand the structure may be different from that to contract the structure.

3.3. *The process of cascade of contraction followed by expansion (C-CE)*

During this process there is a possibility for the structure to either disappear or disintegrate and then disappear, depending upon its original morphological condition. If a force of size $S_{(n)}$ vanishes the structure under the preceding subprocess of contraction, there will be no structure to be expanded. But under the contraction of a structure by a force of size $S_{(n-1)}$, a small portion of the structure will remain to be expanded where the shape of the force influences the resulting structure. This process is represented as

$$\begin{array}{c} (M \ominus S_1) \oplus S_1 \neq \varnothing \\ (M \ominus S_2) \oplus S_2 \neq \varnothing \\ \hline (M \ominus S_{(n-1)}) \oplus S_{(n-1)} \neq \varnothing \\ (M \ominus S_{(n)}) \oplus S_{(n)} = \varnothing \end{array} \tag{10}$$

In the above representation, the origin and direction of force in successive processes of C-CE are identical. Hence they are not represented. The process of C-CE by means of a force $S_{\text{hex}(n-1)}$ results in a transformed structure geometrically similar to the shape of the force, S_{hex}. In fact the resultant structure will depend upon the characteristics of succeeding force. The following are some of the morphological notations which explain this process in a better way.

$$(M \oplus S_{\text{hex}(n-1)\rightarrow}) \oplus S_{\text{hex}(n-1)\rightarrow} \neq \varnothing; \quad [M \ominus S_{\text{hex}(n-1)\rightarrow}] \oplus S_{\text{hex}(n-1)\uparrow} \neq \varnothing \tag{11}$$

$$\left.\begin{array}{l} S_{\text{hex}} = [(M \ominus S_{\text{hex}(n-1)\rightarrow}) \oplus S_{\text{hex}(n-1)\rightarrow}]; \\ S_{\text{hex}(n-1)\uparrow} \equiv \{[M \ominus S_{\text{hex}(n-1)\rightarrow}] \oplus S_{\text{hex}(n-1)\uparrow}\} \end{array}\right\} \tag{12}$$

The change in any of the characteristics of the force during the sub-processes will play a major role in structural transformations. Though there is geometrical similarity between the forces in the cascade process, a change in the origin of the force between the subprocesses leads to deformation of the resulting structure.

3.4. *Sample study: an ideal case*

This sample study aims towards a better comprehension of the impact of force and the process on the structural stability. A hexagonal force (figure 4 (*c*)) is used to transform an elliptical structure (figure 6 (*a*)) following two specific morphological processes, continuous expansion and C-CE. The sequence of structures at respective degrees of expansion and C-CE till both the structure and the force become

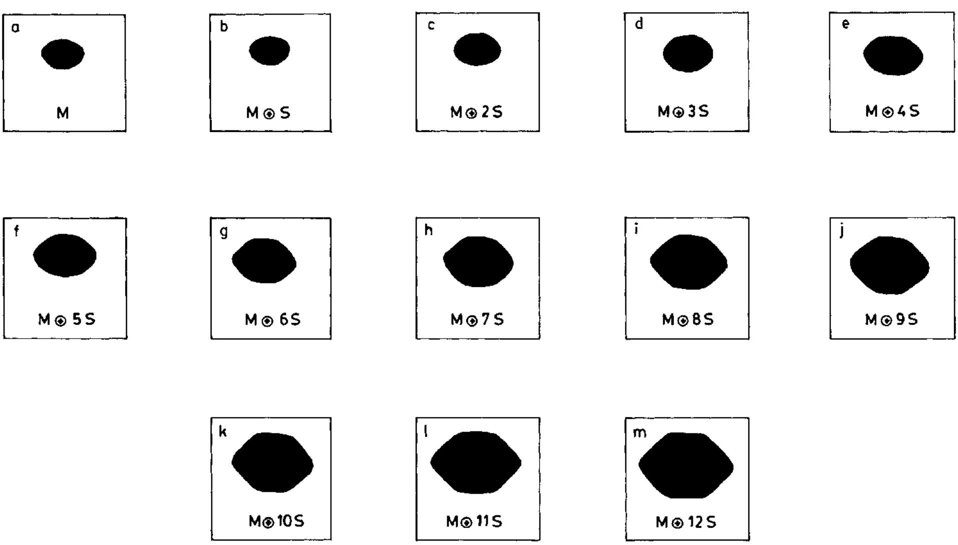

Figure 6 (*a–m*). The sequence of structures at respective degree of expansion.

geometrically similar, is shown in figures 6 (*b–m*), and 7 (*a–f*) respectively. The geometrical similarities between force, original structure and all the resulting transformed structures from these processes are shown in table 1. Based on these results it can be deduced that the hexagonal force is geometrically similar to certain transformed structures as the ratios of their corresponding elements (Strahler 1958) were found to be similar.

3.5. *Laws of structures*

Structures will undergo contraction, expansion, expansion followed by contraction (C-CE), or contraction followed by expansion (C-CE) or any combination of these processes. Based on the properties of structures when subjected to perturbation by any force, five laws of structures have been proposed as follows:

1. A structure reaches the state of convergence during the process of continuous expansion by a force of size more than that of the structure.
2. During the process of continuous contraction by a force of size of more than that of structure, the structure either disappears or disintegrates and then disappears.
3. A Euclidean type of structure will not undergo any change under the process of C-EC. However, there will be a variation in the transformed structure if any of the characteristics of force to expand is different from that to contract.
4. Under the process of C-CE, if the cumulative force acting upon the structure is less than the size of the structure, the transformed structure is geometrically similar to the force. As long as the structure does not disintegrate during this process, if the force to contract is different from that to expand, the morphology of the resultant structure depends upon the succeeding force and the structure remaining just before it gets vanished during the subprocess of contraction. The structure disappears if the cumulative force is more than the structure.
5. Under any process, when both structure and force are geometrically similar, and also the cumulative force does not dominate the structure, the transformed structure will be geometrically similar to both original structure and force.

Based on these laws, a critical point can be defined for the expansion and the cascade processes. ***The critical point is the iteration number at which the structure reaches the state of convergence. This point depends upon the process, characteristics of force, and the original structure.***

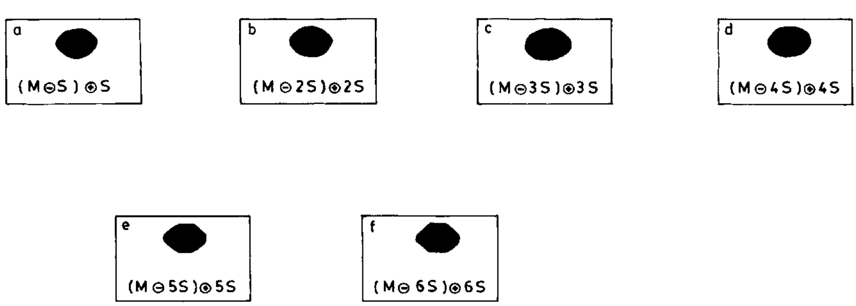

Figure 7 (*a–f*). The sequence of structures of respective degree of the process, i.e., cascade of contraction-expansion.

Table 1. Shows geometrical similarities between force and transformed structure, convergence state, and critical points.

Process	Morphological expression	Physical appearance of structure	Geometrical similarity between force and the transformed structure ratios of corresponding elements					Convergence state	Critical point
			r'/r	p'/p	P'/P	h'/h	l'/l		
Continuous expansion	$M \oplus S$	Ellipse	1·92	1·92	1·63	1·55	1·52	—	
	$M \oplus 2S$	Ellipse	2·2	2·2	1·83	1·29	1·71	—	
	$M \oplus 3S$	Ellipse	1·83	1·83	2·04	1·83	1·9	—	
	$M \oplus 4S$	Ellipse	2·05	2·05	2·24	2·00	2·09	—	
	$M \oplus 5S$	Ellipse	2·5	2·5	2·45	2·17	2·29	Reaching	
	$M \oplus 6S$	Hexagon	2·36	2·36	2·65	2·36	2·48	Reached	6th iteration
	$M \oplus 7S$	Hexagon	2·63	2·63	2·86	2·6	2·67	Reached	
	$M \oplus 8S$	Hexagon	2·71	2·71	3·06	2·70	2·87	Reached	
	$M \oplus 9S$	Hexagon	2·98	2·98	3·26	2·92	3·06	Reached	
	$M \oplus 10S$	Hexagon	3·17	3·17	3·46	2·70	3·25	Reached	
	$M \oplus 11S$	Hexagon	3·72	3·72	3·67	3·20	3·45	Reached	
	$M \oplus 12S$	Hexagon	3·61	3·59	3·87	3·66	3·65	Reached	
Contraction followed by expansion	$(M \ominus S) \oplus S$	Ellipse	1·51	1·51	1·44	1·49	1·53	—	
	$(M \ominus 2S) \oplus 2S$	Ellipse	1·52	1·52	1·42	1·46	1·51	—	
	$(M \ominus 3S) \oplus 3S$	Ellipse	1·52	1·52	1·46	1·51	1·63	—	
	$(M \ominus 4S) \oplus 4S$	Ellipse	1·52	1·52	1·46	1·48	1·51	—	
	$(M \ominus 5S) \oplus 5S$	Hexagon	1·53	1·53	1·55	1·54	1·54	Reached	
	$(M \ominus 6S) \oplus 6S$	Hexagon	1·54	1·54	1·55	1·54	1·54	Reached	5th iteration

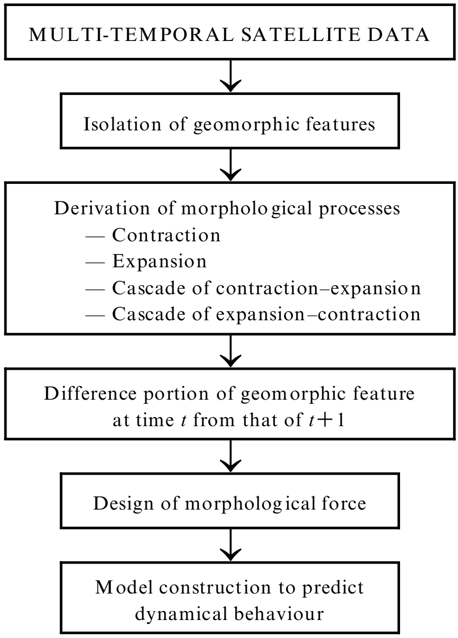

Figure 8. Schematic representation of the procedures that can be followed in the modelling of morphological dynamics using multi-temporal satellite data.

4. Geomorphic evolution modelling: a scope

Many geomorphic features are prone to changes during morphological processes. Such changes can be studied and better monitored by analysing the high resolution multi-temporal satellite data. The sequential steps to model morphological dynamics have been shown in the schematic representation (figure 8).

4.1.1. *Evolution of lake morphology*

In the natural phenomena, the endogenic and exogenic forces show an impact on lake morphology. For instance, a hypothetical representation of the lake morphology at peak summer is shown in figure 9. This figure also shows the intermediary evolution phase of the lake as time progresses towards the peak monsoon season. It is also apparent that the morphological process that the lake has undergone is continuous expansion. In order to design the morphological force that is responsible for the continuous expansion process (peak summer to peak monsoon season), the difference portion between the lake at time t (M^t) and that of the lake at time $t+1$ (M^{t+1}) is considered. To find out the best fitting morphological force, the difference portion needs to be investigated with a set of pre-defined morphological forces as detailed in §2. For the cycle of hypothetical lake evolution process from peak summer–peak monsoon–peak summer (figure 9), the ensuing process is C-EC. This process can be represented mathematically as

$$[\underbrace{(M \oplus S \oplus S \oplus S)}_{\textit{Expansion phase}} \underbrace{\ominus S \ominus S \ominus S}_{\textit{Contraction phase}}] \tag{13}$$

The reverse process can also be visualised as two successive phases (peak monsoon–peak summer–peak monsoon), i.e. cascade of contraction-expansion (C-CE). The study of nonhomogeneous nature of these basic morphological processes sheds

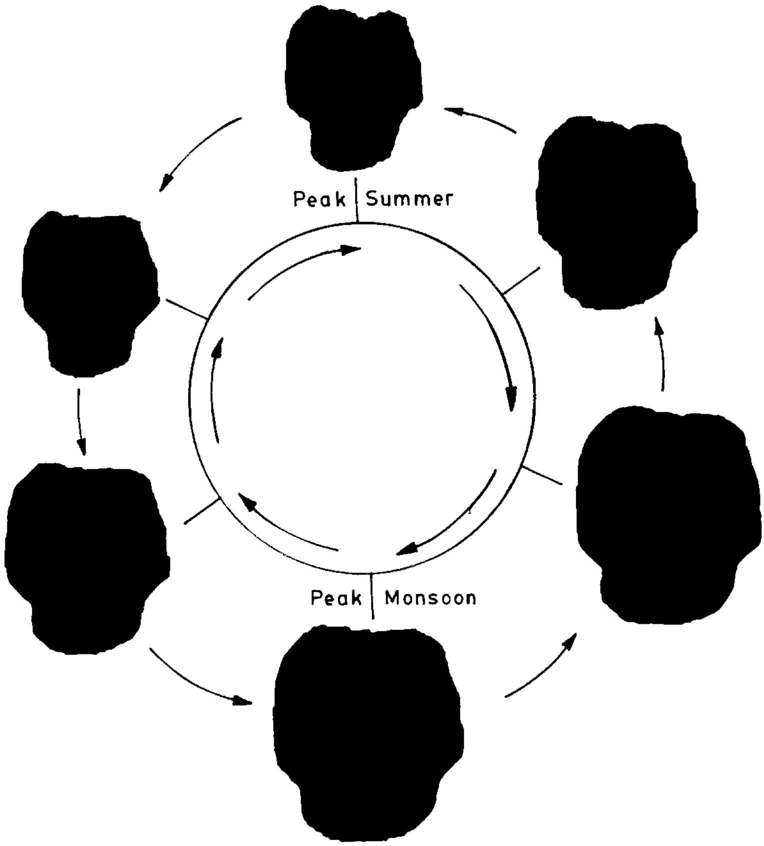

Figure 9. A qualitative representation of morphological evolution of a lake from peak summer–peak monsoon–peak summer (clockwise direction, C-EC process)–peak monsoon–peak summer (anticlockwise direction, C-CE process).

light on the study of the dynamical process in the natural lake evolution. In non-ideal cases, there may be dissimilarity in the characteristics of the cumulative force from one phase to the other. A discrepancy at any stage in the force, designed by considering the feature at successive time intervals, may be attributed to the change of phase.

4.1.2. *Process of meander separation as oxbow lake*

The characteristics of the force may vary in accordance with the dynamical process of the geomorphic system. A hypothetical study of the evolution process of a geomorphic feature, for instance, a meandering process leading to the formation of oxbow lake is carried out by the process of C-CE (figure 10 (a, b)). During this process, the formation of an oxbow lake which gets separated from the main course of the river is also clearly seen in figure 10 (b).

5. Modelling of morphological dynamics of lake: a qualitative study

The morphological conditions of a geomorphic feature play a significant role in predicting its morphological behaviour. Since most geomorphic features have irregular shapes, descriptive analysis is of limited use. In this section, an attempt is made

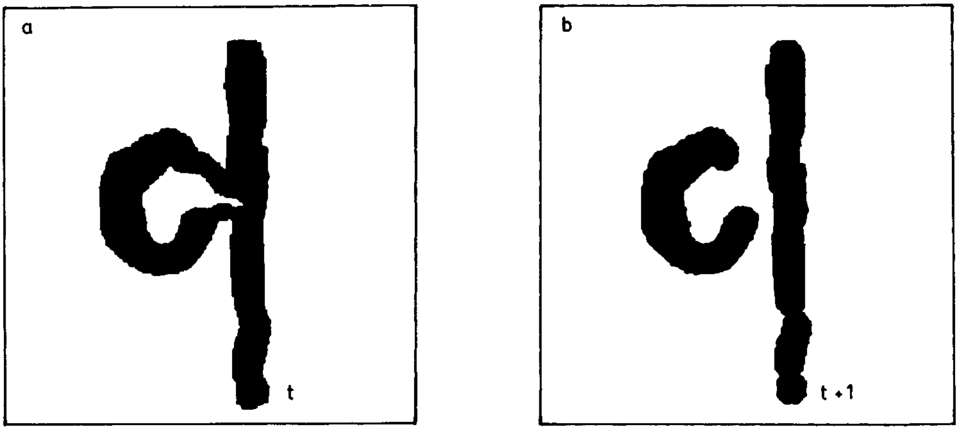

Figure 10. The process of meandering separation by mathematical morphological process. Its diagrammatic representation from time t to $t+1$.

to analyse the geomorphic evolution process systematically through mathematical morphological transformations. From a topological point of view, a circular type of system is more stable than the system with a non-standard morphological form. A geomorphic system at equilibrium state is more stable than that at inequilibrium state. The morphological stability of a geomorphic system at equilibrium state may be defined by the morphological behaviour of the system when it is subjected to a small perturbation. A system is said to be stable if it returns to its original state and unstable if it continues to move away from equilibrium state as a result of a perturbation caused by a homogeneous morphological force. Thus, the qualitative analysis has great significance to understand the geomorphic structural dynamics. In this section, a qualitative study has been presented where a first-order difference equation has been taken as the basis. This analysis reduces the complex system to a simpler form that captures the important features of the original system.

Logistic map analysis: The following is a maiden attempt to model morphological dynamics which follow an ideal condition. The variation in the structure under specific transformation can be quantified by constructing a one-dimensional (1-D) map (logistic map). As the theory of one-dimensional maps constructed by iterating the first order difference equation is well established (May 1976), it will be useful if an appropriate 1-D map can be constructed from the structure under study.

A typical logistic map is constructed by iterating the following first-order difference equation proposed in the context of ecology (May 1976)

$$X_{t+1} = \lambda X_t(1 - X_t) \tag{14}$$

This first-order difference equation was used in many studies to quantify several natural processes (May 1976, Jenson 1987, Sagar and Rao 1995 a, b). In this equation the strength of nonlinearity, λ, gives the entire description of the changing population. For a higher value of X_t, the expression $(1 - X_t)$ reduces the output value and *vice versa*.

The essential parameters to construct a logistic map are the initial value, X_t, represented in a normalised scale, and the strength of nonlinearity, λ. In this study, instead of the population X_t and X_{t+1}, the fractal dimensions (Mandelbrot 1982) in normalised scale α_τ and $\alpha_{\tau+1}$, useful to quantify the degree of irregularity of the generated shapes before and after every process and iteration, are considered. To represent the fractal dimensions in a normalised scale, the topological dimension (D_T) is subtracted from the fractal dimension (D).

$$\alpha = D - D_T \tag{15}$$

If a structure transforms itself by following a specific rule, the resultant transformation at time $t+1$ can be determined from the structure at initial time t. This relation can be shown mathematically as

$$\alpha_{t+1} = f(\alpha_t) \quad \text{or } \alpha_{t+1} = f_\lambda(\alpha_t) \tag{16}$$

$$\alpha_{t+n} = f^n(\alpha_t) \quad \text{or } \alpha_{t+n} = f_k(\alpha_{t+(n-1)}) \tag{17}$$

where function f is defined in one-dimensional space and λ is the magnitude of variation from time t to $t+1$. This function depends upon λ which can be incorporated as a single description to predict the behaviour of a structure.

Considering a specific process, the future structure may be predicted through intensive studies of the first-order difference equation. This study mainly depends upon the computation of the strength of nonlinearity (λ), which varies with the structure, and type of force (disturbance) acting on the structure. For small values of λ, the system is stable (i.e. $\alpha_{t+1} = \alpha_t$ and so on). As λ increases, the system moves away from its stable state. An equation to compute the strength of nonlinearity in structures may be derived by considering the structure at times t, and $t+1$. To fit a logistic map to a set of observations α_t, $\alpha_{t+1}, \ldots$, a plot between α_{t+1} and α_t is necessary to estimate the value needed to fit the curve $y = \alpha_t(1-\alpha_t)$.

Attracting to a fixed point: In this section a deterministic approach has been followed to model the dynamics of transcendentally generated fractal lake under specified morphological transformation. To explain and quantify the entire morphological evolution process of a hypothetical lake going towards extinction, a fractal lake (figure 11 (*a*)) is allowed to undergo the C-CE process iteratively by means of a octagonal morphological force up to four cycles. Iterations beyond the fourth cycle are not considered to construct the model as the fractal lake is getting vanished. Figures 11 (*b–d*) are after respective cycles of C-CE process (e.g., contraction phase of lake evolution from peak monsoon to peak summer, and expansion phase from peak summer to peak monsoon) by means of an octagonal morphological force (figure 4 (*b*)). The textural and structural variations in the sequence of the transformed fractal lakes are due to the increase in the size of the force during this cascade process. As the force is smaller in the initial phase of transformation, a textural variation is observed while in the latter phase a structural variation is observed due to increase in the force. The deformation in the transformed fractal lakes from iteration to iteration is quantified in terms of fractal dimension (table 2) computed through box counting method proposed elsewhere (Feder 1988). The computed fractal dimensions for the four transformed structures obtained from the respective cycles are 1·53, 1·38, 1·36, and 1·34 (table 2). From the normalised fractal dimensions, α, at discrete time periods, the strength of nonlinearity, λ, is estimated as 1·53. A logistic map (figure 12) is constructed for this process using the strength of nonlinearity and the initial normalised fractal dimension. The trajectory of the logistic map is traced to predict the successive values of α. The predicted fractal dimensions ($\alpha + D_T$) of the fractal lake inferred from the logistic map are close to computed

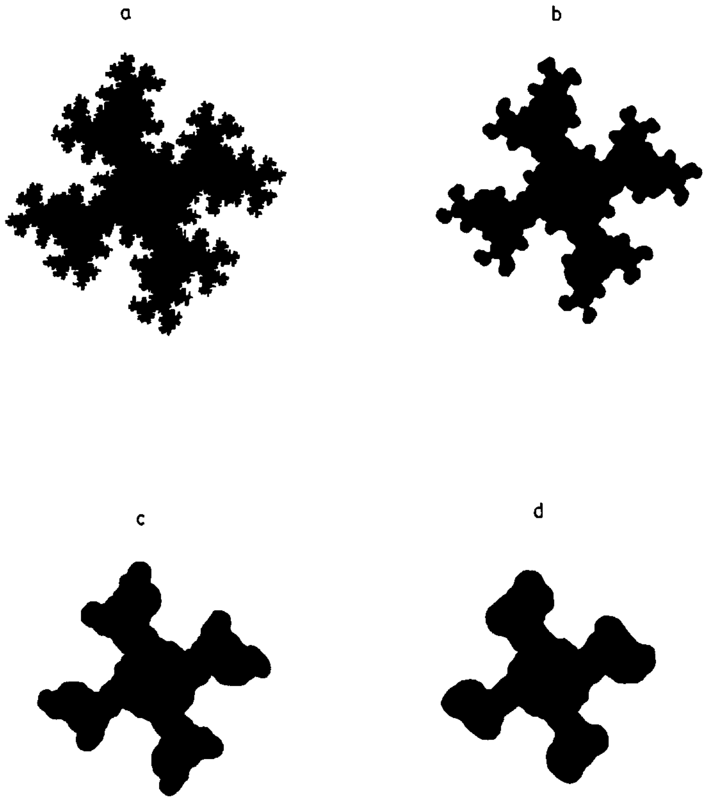

Figure 11. (*a*) Fractal lake, (*b*) lake after one cycle of C-CE, (*c*) lake after two cycles of C-CE, (*d*) lake after three cycles of C-CE.

Table 2. Morphological dynamics: computed and predicted fractal dimension values.

	Computed fractal dimension and α values		Predicted fractal dimension and α values	
Morphological process	Fractal dimension	α	Fractal dimension	α
$M \ominus S \oplus S$	1·53	0·53	1·53	0·53
$M \ominus S \oplus S$	1·38	0·38	1·386	0·386
$M \ominus S \oplus S$	1·36	0·36	1·36	0·36
$M \ominus S \oplus S$	1·34	0·34	1·35	0·35

fractal dimensions for the respective cycles (table 2). Though this model is qualitative, this approach where mathematical morphology, fractals, and chaos theory are integrated, helps to provide cogent models for certain geomorphic evolution processes.

As an example, a deterministic approach to model morphological changes in the lake's outline has been given, which may be of use to model the morphological dynamics of a geomorphic feature that undergoes morphological changes over time periods. In this, the use of logistic equation is shown as an optional one. However, there is every possibility to compute the strength of nonlinearity of several parameters

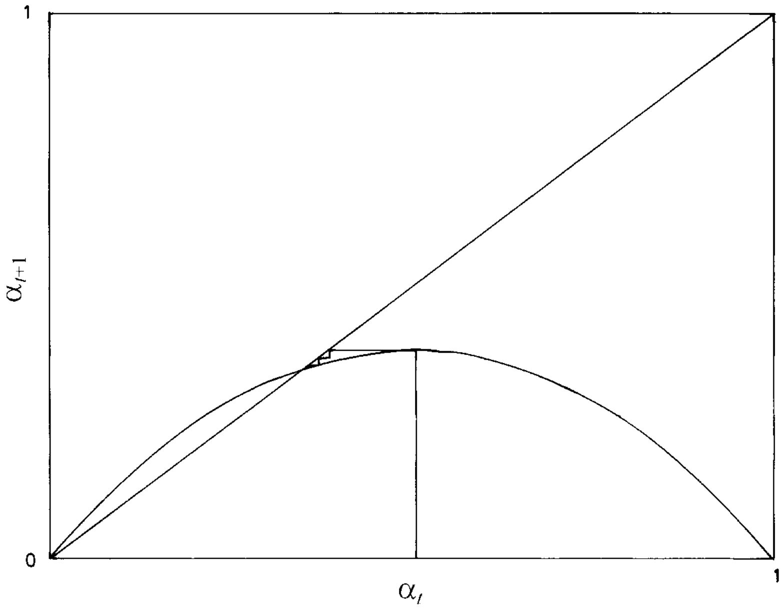

Figure 12. The representation of dynamical changes in the morphology of evolving lake through logistic map. $\lambda = 1{\cdot}53$.

of a feature. In the present paper, the parameter that is made use of is the fractal dimension of perimeter of a transcendentally generated lake that is transformed according to a specific morphological process by a defined morphological force.

To model the morphological dynamics of real world geomorphic structure, the best source of information is the various types of data acquired by remote sensing satellites in multitemporal domain. The other way of modelling the morphological dynamics is perhaps by recording the morphological changes episodically or continuously using GPS's. Further, the scope of the work in modelling the dynamics of real world geomorphic data using multi-temporal satellite data by following the concepts of mathematical morphology is foreseen positively.

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