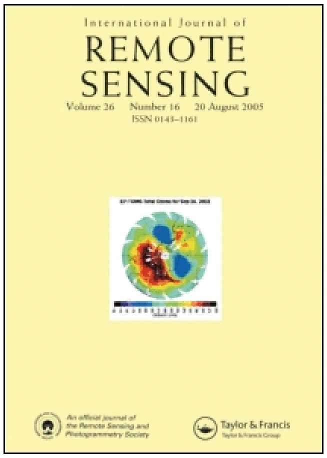

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Applications of mathematical morphology in surface water body studies

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Abstract. Some possible applications of mathematical morphological transformations in computing the basic measures of surface water bodies are presented. Sixteen water bodies are extracted from SPOT PLA data, and the algorithm developed, based on mathematical morphological concepts, is tested to compute their basic measures.

1. Introduction

In mathematical morphology, the concept of image functions in terms of basic measures, area, perimeter, centroids, and directional diameters (Ellias and Weibel 1967, Meyer 1980, Giardina and Dougherty 1988). Kulpa (1977, 1983) used algorithms based on arithmetics to compute basic measures. But the computation of these basic measures of any shape by planimetric methods is tedious, time consuming and the results are only approximate. Also, the algorithms that depend upon real arithmetics are plagued by the inadequacies resulting from the so-called finite length register effects (Sinha and Giardina 1990). Davis (1986) opines that the trapezoid approximation, a computational method for the determination of a centroid is not simple. The investigations of Lantuejoul (1982), Meyer (1980), Chermant *et al.* (1984) have proved the potentiality of mathematical morphology applications for the determination of shape parameters in various disciplines.

Therefore, in the present investigation certain mathematical morphological transformations *erosion* and *hit or miss transformation* (Serra 1982) and erosion dependent *grassfire transformation* (GFT), are tested to compute the basic measures of surface water bodies in discrete domain.

2. Mathematical morphological transformations

Mathematical morphology based on set theoretic concepts is a particular approach to the analysis of geometric properties of different structures. The main objective is to understand the geometrical properties of a binary image of a natural feature, consisting of water bodies, refer to as grain, and no-water body, refer to as pore, by investigating its microstructures with various forms known as 'structuring elements', following the concept of Serra (1982). In this binary format '1's indicate parts of the water body and '0's show parts of no-water body. The shape of the structuring element depends upon the orientation of grain or its parts. In the present context an eight connected square grid is considered as a structuring element.

The geometrical properties of a binary image, possessing water bodies and no-water bodies, are subjected to the morphological functionals erosion, grassfire, and hit or miss transformations. A discrete binary image, M, is defined as a finite subset of Euclidean two-dimensional space, R^2.

The following morphological operations are used for developing an algorithm to compute basic measures.

Translation: Let D be a subimage of the Euclidean plane, R^2. The translation of the image D by a point, x, in R^2 is denoted by $D+x$ and defined as:

$$D+x=\{d+x\colon d\in D\} \tag{1}$$

Dilation: Let M and S be subimages of Euclidean plane, R^2. The dilation of an image, M, with structuring element, S, is denoted by $\mathscr{D}(M,S)$ and defined as:

$$\mathscr{D}(M,S)=M\oplus S=\cup_{s\in S}M+s=\{x\colon(-S+x)\cap M\neq\phi\} \tag{2}$$

Erosion: Let M and S be subimages of Euclidean plane, R^2. The erosion of an image, M, with structuring element, S, is denoted by $\xi(M,S)$ and defined as:

$$\xi(M,S)=M\ominus(-S)=\cap_{s\in S}M+s=\{x\colon S+x\subseteq M\} \tag{3}$$

where $-S=\{-s\colon s\in S\}$, i.e., S rotated 180° round the origin. Hereafter $-S$ is represented as $\check{S}$. Now $\xi(M,S)=M\ominus\check{S}$. Two consecutive erosions can be defined as follows:

$$(M\ominus\check{S})\ominus\check{S}=M\ominus(\check{S}\oplus\check{S})=M\ominus 2\check{S} \tag{4}$$

2.1. *Boundary extraction*

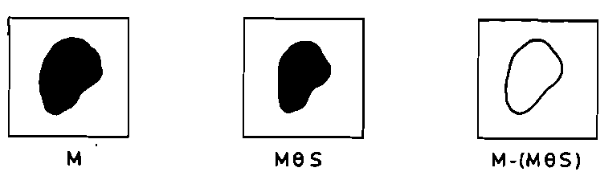
The boundary (figure 1 (*c*)) of any object, M, can be extracted by subtracting the eroded object, $\xi(M,S)$, (figure 1 (*b*)) from the original object (figure 1 (*a*)). This can be mathematically represented as,

$$\text{Boundary}=M-\xi(M,S)=\{\text{x}\colon \text{x}\in M \text{ and } \text{x}\notin\xi(M,S)\} \tag{5}$$

2.2. *Centroid determination*

According to the grassfire transformation (GFT) of Meyer (1980) a fire starts simultaneously at all points along the boundary of a grain, and propagates at uniform speed towards the centroid of the grain. The centroid of any shape can be identified, irrespective of its complexity, with a higher degree of precision, provided the following conditions are satisfied (figure 2). Two shapes, simple with uniform outline and complex with the corrugated outline, are considered to explain the process of grassfire transformation to locate the centroid.

If an object of simple form, can be assumed to vanish at n consecutive erosions, as shown in figure 2 (*b*), its centroid can be identified at the $(n-1)$th erosion (figure



(*a*) (*b*) (*c*)

Figure 1. Morphological operations, (*a*) original image in binary format, (*b*) eroded set, and (*c*) subtracted set from (*b*), boundary.

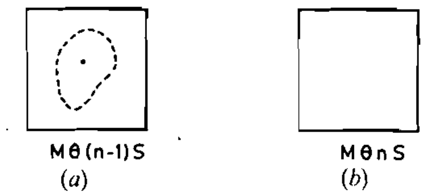


Figure 2. Grassfire transformation, (*a*) set after $n-1$ number of erosions. (*b*) set after n number of erosions, empty set.

2(*a*)). The consecutive n erosions of an object, M, with the structuring element, S, can be represented as follows:

$$\begin{aligned} M\ominus n\check{S} &= (\ldots(((M\ominus\check{S})\ominus\check{S})\ominus\check{S})\ldots\ominus\check{S})n \text{ times} \\ M\ominus(n-1)\check{S} \neq \phi &: \qquad M\ominus n\check{S}=\phi \end{aligned} \tag{6}$$

The single pixel remaining, after operation $M\ominus(n-1)\check{S}$ is considered as the centroid of the shape (figure 2(*a*)).

For complex shapes, when more than one pixel remains; they have to be joined. The resultant shape may be a straight line, a triangle, a square, a parallelogram or any possible polygon which may be termed a 'secondary set' or 'secondary grain' or 'connex components' (figure 3(*a*)). The secondary sets have to be blocked in, and grassfired once again.

While applying GFT, a complex shape (figure 3(*c*)) may be separated into k different connected components, C_1, C_2, ..., C_k, after $m(1<m<n)$ consecutive erosions (figure 3(*b*)), where $C_i \subseteq M$, $1 \leq i \leq k$ (Definer 1972). Treating each connex component as a set, the sets are grassfired again to determine the local centroids (figure 3(*c*))

$$C_i\ominus nS=\phi \quad \text{and} \quad C_i\ominus(n-1)S\neq\phi, \quad \text{where} \quad 1<i<k \tag{7}$$

$C_i\ominus(n-1)S$ is the centroid of that set C_i (figure 3(*c*)) which is a subset of M. The polygon obtained by connecting local centroids, is filled as a solid block and termed a tertiary set (figure 3(*d*)). The tertiary set is grassfired, once more to find out the actual centroid of the original shape (figure 3(*e*)).

2.3. *Area computation*

In step 3 the structuring element, S, of size 1×1 (i.e., a single image pixel) is selected and convolved over the image. Whenever a structuring element fits (i.e.,

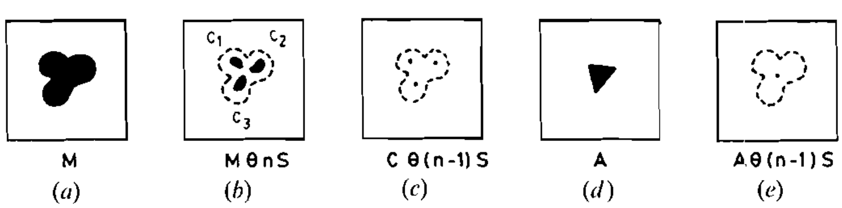


Figure 3. Grassfire transformation on complex shapes, (*a*) original image in binary format (M), (*b*) set after m number of erosions, three connected components (C_1, C_2, and C_3) can be identified, (*c*) set after $n-1$ number of erosions, three local centroids of three connex components, (*d*) secondary set after adding local centroids, (*e*) secondary set after $n-1$ number of erosions.

Figure 4. SPOT PLA digital data (512 × 512).

$S \subseteq M$) within the image, it will be considered as a *hit* or a *miss* (i.e., $S \subseteq M$). Whenever the structuring element hits the microstructure of the set, it is counted as an area element.

$$\text{Area of the Set considered} = \text{Total Hits scored} \tag{8}$$

The algorithm thus developed has been tested successfully on water bodies extracted from satellite digital data.

3. Case study

The mathematical morphological techniques were applied on surface water bodies extracted from the digital data sets of small area (5 km × 5 km), acquired through SPOT (PLA mode) digital data of December 1990 (figure 4), to compute basic measures. The flowchart shows the sequence of procedures followed in this

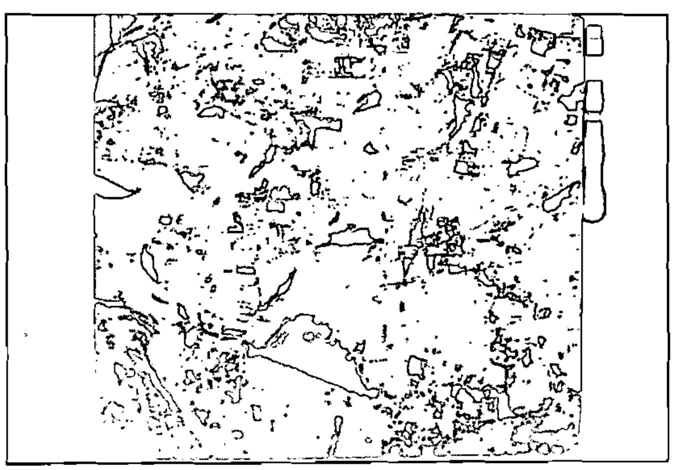


Figure 5. The image after applying threshold values.

Figure 6. The thresholded image after editing, showing water bodies and no-water body region.

work. The water bodies were extracted using thresholding (figure 5) and image editing techniques (figure 6). The boundaries of the water bodies were extracted by subtracting the eroded water bodies from the original water bodies (figure 7). The sample consisted of 16 water bodies each with a minimum area of $100\,m^2$ as SPOT (PLA) digital data possess $10m \times 10m$ resolution (figure 6).

3.1. *Application of the developed algorithm*

A program was developed for the algorithm discussed in the previous sections to compute certain basic measures of the water bodies (figure 9). The area and perimeter of all the water bodies were computed by applying a hit or miss transformation (HMT), and the centroids of all water bodies were determined by applying a grassfire transformation (GFT). Based on the centroids identified for

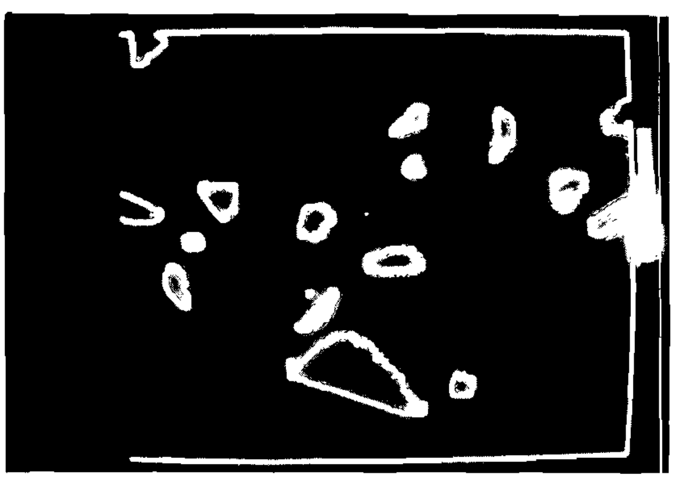


Figure 7. The boundaries of surfaçe water bodies.

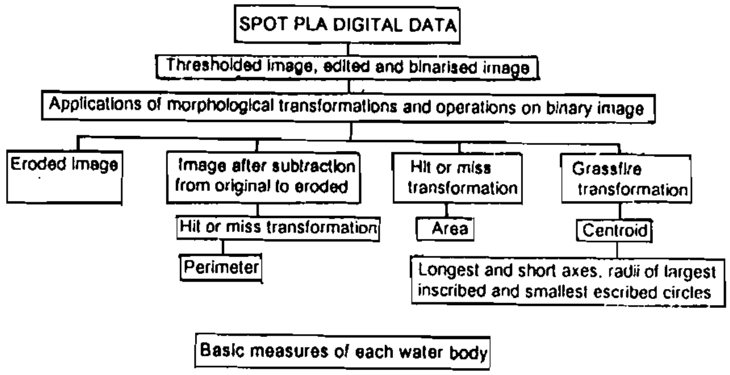


Figure 8. Flowchart shows procedures followed in this work.

every water body, the longest axis, width, together with the radii of the largest inscribed and the smallest inscribed circle were computed.

The basic measures of sixteen water bodies, extracted from these satellite digital data, and their dimensionless ratios are presented in tables 1 and 2. The dimensionless properties were computed by using the formulae of Folk and Rayner (see Davis 1986, page 342). The centroid, being an important parameter in many aspects, is determined through successive erosions. The proposed method is better than several existing methods. From the centroid the distances can be measured to every pixel on the boundary, in 360° rotation, to represent the water body, which is in closed form, into a single Fourier line. Thus outline forms of the water body can be characterized in the Fourier domain. The computation of perimeters and area are also of use to compute the shape ratios. The relations among these primary measures can also be shown.

4. Conclusions

The mathematical morphological transformations are useful for the computation of basic measures of any type of surface water body, and avoid the constraints of

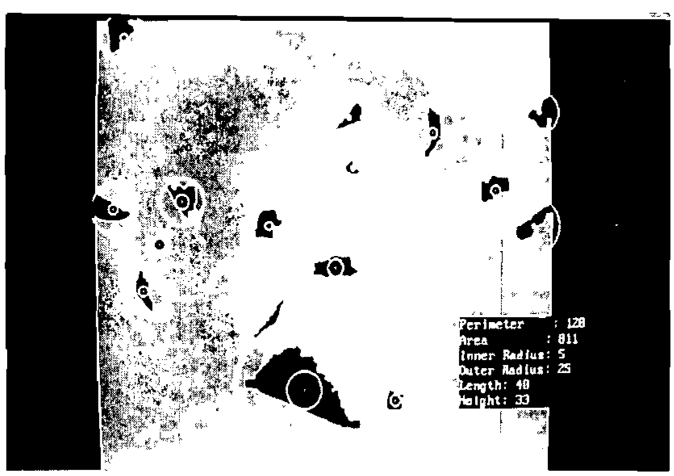


Figure 9. The image after applying mathematical morphological transformations.

Table 1. Basic measures of sixteen water bodies. (All the units are in pixels.)

Water body number	Perimeter	Area	Inner radius	Outer radius	Length	Width
1	188	1629	4	34	54	45
2	131	1103	8	29	42	40
3	152	1364	6	25	47	23
4	62	393	7	18	19	15
5	161	1572	8	24	39	34
6	476	8532	23	78	134	90
7	141	1400	5	20	37	33
8	181	2341	2	28	45	39
9	164	1458	10	29	57	28
10	138	1335	3	23	37	32
11	82	586	3	12	21	19
12	99	825	7	16	26	24
13	178	1556	6	30	57	24
14	199	1251	5	22	37	36
15	204	1654	5	31	47	45
16	180	1293	7	29	48	36

manual methods. Hydrologists may prefer to adopt these mathematical morphological concepts to compute basic measures, rather than the use of algorithms that rely on real arithmatics.

The primary measures are used to study irrigation tanks in a temporal domain. The areal extent can be computed, facilitating hydrologists to cary out temporal analysis at a faster rate. In particular, temporal changes in binary images can be studied. A binary image, consisting of a number of water bodies, can be subtracted

Table 2. Dimensionless parameters of sixteen water bodies.

Water body number	Circularity ratio	Elongation ratio	Form ratio	Compactness coefficient	Thinness ratio
1	0·18	0·83	0·56	1·73	0·58
2	0·24	0·95	0·63	1·24	0·81
3	0·24	0·49	0·62	1·35	0·74
4	0·41	0·79	1·09	0·78	1·29
5	0·24	0·87	1·03	1·31	0·76
6	0·15	0·67	0·48	2·11	0·47
7	0·28	0·89	1·02	1·11	0·89
8	0·29	0·86	1·16	1·47	0·90
9	0·22	0·49	0·45	1·11	0·68
10	0·28	0·86	0·98	1·47	0·88
11	0·35	0·90	1·33	1·14	1·10
12	0·34	0·92	1·22	0·91	1·06
13	0·20	0·42	0·48	0·95	0·62
14	0·13	0·97	0·92	1·62	0·32
15	0·16	0·95	0·75	2·00	0·50
16	0·16	0·75	0·56	1·99	0·50

from the same image of another season. This aids analysis of directions and changes in the areal extent of the water bodies. These primary measures can be used to compute various shape ratios (dimensionless properties) empirically. These studies can be extended to any unit of spatial shape. This computationally simple algorithm also works well on complicated shapes.

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