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Technical note

Morphological operators to extract channel networks from digital elevation models

B. S. DAYA SAGAR†, M. VENU and D. SRINIVAS

Centre for Remote Sensing & Information Systems, Department of Geo-Engineering, Andhra University, Visakhapatnam 530 003, India; e-mail: cysbsds@nus.edu.sg

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Abstract. The automated extraction of drainage networks includes generation and processing of digital elevation models (DEMs) obtained from the remotely sensed data having stereo viewing capability. The latter aspect generally aims to extract terrain features such as elevation contours and channel networks. In this technical note, the application of morphological operators to extract channel networks from the digital elevation model is described. The methodology is illustrated using a transcendentally generated DEM that bears the spatially distributed regions in grey levels, assumed as the regions of topographic reliefs and the V-shaped crenulations in successive elevation contours. The authors conclude that the adaptation of this approach to extract channel networks from DEM data is straightforward and is simple both algorithmically and computationally.

1. Introduction

One of the most interesting terrain features is channel networks. Hitherto, aerial photographs were used to obtain information concerned with the quantitative description of drainage basins and channel networks. These techniques had tremendous advantages over the conventional usage of topographic maps. The work by Bunik and Turner (1971) showed that aerial photographs are often superior to topographic maps for obtaining information. Now, remote sensing satellite sensor data has proved to be much more advanced and inexpensive for obtaining information with more efficiency. To study the quantitative description of channel networks in particular, the satellite sensors with stereo viewing capability are very useful and can be used to generate digital elevation models (DEMs). A DEM is a numerical representation of topographic surfaces. The DEMs, which are arrays of numbers that represent the spatial distribution of terrain altitudes, can be derived indirectly from digitized topographic maps or directly through photogrammetric processing of medium scale, black and white, metric aerial photographs (Franklin 1990). Recently, many algorithms were implemented to generate DEMs using satellite sensor images

†Current address: Centre for Remote Imaging, Sensing, Processing (CRISP), Faculty of Science, The National University of Singapore, Lower Kent Ridge Road, Singapore-119260.

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acquired chiefly by the SPOT (System Probatoire d'Observation de la Terre) and IRS-1C (Indian Remote Sensing) satellite sensors.

The three sequential steps involved in terrain analysis using remotely sensed data are: 1) processing of stereo remote sensing data, 2) generation of digital elevation models from the remotely sensed data having stereo-viewing capability, and 3) extraction of terrain information such as elevation contours and channel networks. All these steps were attempted and established by several researchers (O'Callaghan and Mark 1984, Yuan and Vanderpool 1986, Jenson 1987, Jenson and Domingue 1988, Mark 1988, Martz and deJong 1988, Morris and Heerdegen 1988, Qian *et al.* 1990, Fairfield and Leymarie 1991, Freeman 1991, Tribe 1992, and Chorowicz *et al.* 1992). Although these algorithms address the idea of drainage extraction from DEMs, they are found to be computationally complex. Thus, an efficient procedure of practical interest based on mathematical morphology is introduced as the existing algorithms share several limitations (Fairfield and Leymarie 1991).

1.1. *What do angular points in DEM represent?*

The regions of elevations in DEM that are depicted by grey levels contain several angular points. These angular points, the V-shaped portions of the elevation contour in 2-dimensional space which are referred to as crenulations, indicate the presence of channels (Morisawa 1957).

1.2. *Composition of channel networks*

The positions of the crenulations in the sequential elevation contours determine the complex channel networks (e.g. Driftwood, Nashville, Little Tujunga, Cuny Table West quadrangle maps of US Geological Survey, Strahler 1964). The number of channel branching orders depends on the overall structure of the basin. Depending on the structural composition of the basin, the channel networks of lower order will bifurcate and the channel networks of next higher order will be formed. After flowing to a certain distance, the two first order streams join to form a second order stream segment (Strahler 1964), the flow path of which is another crenulation wider than that of the preceding order crenulation. Thus, the widths of the crenulations of contours of higher elevation are smaller than those of the lower elevation contours. It indicates that fewer higher orders are present than the lowest or lower orders. The union of all possible crenulations in the maximum number of contours produces an aggregate of the channel network. The following sections present a detailed procedure, based on mathematical morphological operators to extract channel networks from digital elevation models along with a sample study and conclusions.

2. Methodology

The methodology proposed in this section is robust enough to extract channel networks from the DEM generated from the remotely sensed data. The procedure to extract drainage networks is twofold:

- Threshold decomposition of the DEM.
- Isolation of drainage subsets using a morphological procedure from all the thresholded regions of the DEM.

2.1. *Threshold decomposition*

The transformations defined for binary images are equally good for a grey scale image (e.g. DEM) (Serra 1982, Wendt *et al.* 1986, Serra and Vincent 1992) taking their grey level values in $\{0, 1, ..., i-1\}$, it suffices to consider the successive thresholds $T_k(I)$ of I, for $k=0$ to $i-1$.

$$T_k(I) = \{p \in D_I \mid I(p) \geq k\} \tag{1}$$

where k ranges from 1 to 16 in the sample study given in §3. p is a neighbour of q if and only if (p, q) belongs to a discrete grid G that is a subset of $Z^2 \times Z^2$. D_I is the finite rectangular subset of the discrete plane Z^2 into a discrete set $(0, 1, ..., i-1\}$ of grey levels. In the present study, the square grid in 4-connectivity is used. The ensuing expression shows the inclusion relationship.

$$T_k(I) \subseteq T_{k-1}(I) \qquad \forall k \in [1, i-1] \tag{2}$$

Figure 1 exemplifies the inclusion relationship. Henceforth, the thresholded image, $T_k(I)$, is referred to as A which is a binary image that can admit a value of 0 or 1; often considered as the set of its pixels with value 1. The pile of boundaries of all the thresholded images of the DEM is presupposed as the successive elevation contours or successive frontlines. The crenulations in the boundary of each thresholded image indicate the presence of a channel. The surface undulations and the system of ridges produce the structural composition of the channel morphology. To extract probable drainage channel networks from the thresholded image (A), the procedure is as follows. To clarify this procedure, certain basic mathematical morphological transformations are detailed with the list of symbols and notation. Most of this mathematical formalism and notation is adopted from the works of Lantuejoul (1980) and Serra (1982).

2.2. *Mathematical morphology*

2.2.1. *List of symbols and notation*

Z^2	2-dimensional grid of discrete space
$A, B, M, \ldots$	Subsets of Z^2
$\emptyset$	Empty set
$a, b, m, \ldots$	Elements of vector points of Z^2, i.e. a point in the 2-D space
$a \in A$	Element a belongs to A
$a \notin A$	a does not belong to A
$\subset, \subseteq, \cap, \cup$	Subset, improper subset, intersection, and union
$M \ominus B$	Minkowski subtraction

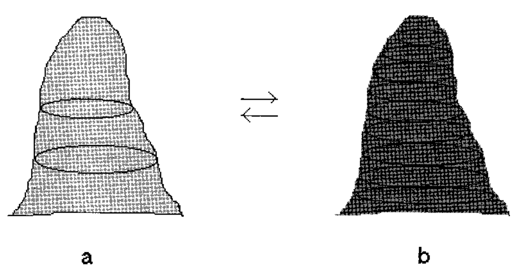

Figure 1. Threshold decomposition of a DEM (*a*) function and mapping, (*b*) pile of sets.

$M \oplus B$	Minkowski addition
$A \ominus \hat{B}$	Erosion of A by B
$A \oplus \hat{B}$	Dilation of A by B
M/A or $\frac{M}{A}$	Set difference between M and A
A^c	Complement of A with respect to Z^2
A_b	Translate of a by vector b, i.e. $\{a: a-b \in A\}$
$\cap_{b\in B} A_b$	Intersection of all the translates of A_b, with $b \in B$
$\cup_{b\in B} A_b$	Union of all the translates A_b, with $b \in B$

One of the approaches to the analysis of geometric properties of different structures is mathematical morphology, which is based on set theoretic concepts. The thresholded discrete binary image, A, obtained from the DEM is defined as a finite subset of Euclidean two-dimensional space, Z^2. The geometrical properties of each thresholded image that contains set (A) and its complement (A^c) are subjected to the morphological functions. The morphological operators may also be considered as working with two images. The image being processed is referred to as the thresholded image which needs to be decomposed into channel subsets, and the other image as a structuring template (B). Each structuring template bears a designed shape that can be thought of as a probe of all the thresholded images decomposed from the DEM. In addition, many structuring templates can be represented by a compact subset of Euclidean space. This enables the constraints which agree with the four principles of mathematical morphological theory, to be imposed on morphological set transformations (erosion, dilation, opening and closing) for precise extraction of topological information from the drainage network extraction point of view. These principles are invariance under translation, compatibility with change of scale, local knowledge and uppersemicontinuity. The three morphological transformations based on Minkowski set addition and subtraction (Hadwiger 1957) used in drainage channel network extraction are dilation to expand, erosion to shrink, and cascade of erosion-dilation.

Dilation: Dilation combines two sets using vector addition of set elements. If A and B are sets in Euclidean space with elements a and b respectively, where $a=(a_1, \ldots, a_N)$ and $b=(b_1, \ldots, b_N)$ being N-tuples of element coordinates, then the dilation of A (thresholded image points) by B (structuring template) is the set of all possible vector sums of pairs of elements, one coming from A and the other from B. The dilation of a decomposed thresholded image of the DEM, A, with structuring template, B, is defined as the set of all points 'a' such that B_a intersects A. It is expressed as

$$A \oplus B = \{a : B_a \cap A \neq \varnothing\} = \cup_{b\in B} A_b \tag{3}$$

Erosion: The erosion of a decomposed thresholded image of the DEM, A, with structuring template, B, is defined as the set of points 'a' such that the translated B_a is contained in A. It is expressed as

$$A \ominus B = \{a : B_a \subseteq A\} = \cap_{b\in B} A_b \tag{4}$$

where $B=\{b: b \in B\}$, i.e. B rotated 180° around the origin. It is worth mentioning here that Minkowski addition and subtraction are akin to the morphological dilation and erosion as long as the structuring template (B) is of the symmetric type (Maragos and Schafer 1986).

Cumulative structuring template (B): The cumulative structuring template can be mathematically represented as

$$B_n = B \oplus B \oplus B \oplus \ldots \oplus B \tag{5}$$

According to the above expression, two consecutive erosions and dilations can be represented as $(A \ominus B) \ominus B = A \ominus B_2$, and $(A \oplus B) \oplus B = A \oplus B_2$ respectively. The structuring template of square shape is shown in figure 2. The dilation followed by erosion is called closing transformation. Cascade of erosion-dilation is called opening transformation. These cascade transformations are idempotent (Serra 1982). However, these transformations can be carried out according to the multiscale approach (Maragos and Schafer 1986) where the size of the structuring template would be increased from iteration to iteration.

2.3. *Decomposition of each thresholded region into drainage subsets*

A drainage network is defined as a line thinned caricature that summarizes the shape, size, orientation and connectivity of the watershed. A drainage network is defined according to equation (9) as the union of all possible angular points isolated from all possible successive front-lines of all the decomposed thresholded regions of the DEM. The possible drainage network from decomposed regions (A), CH(A), viewed as subsets of Z^2 can be defined mathematically as

$$\mathrm{CH}_n(A) = \frac{A \ominus B_n}{[(A \ominus B_n) \ominus B] \oplus B} \quad n = 0, 1, 2, \ldots, N \tag{6}$$

$$\mathrm{CH}(A) = \cup_{n=0}^{N} \mathrm{CH}_n(A) \tag{7}$$

where $\mathrm{CH}_n(A)$ denotes the nth drainage subset of thresholded region (A). In equation (6), subtracting from the eroded versions of A their opening by B retains only the angular points, which are points of the drainage network. The union of all such possible points produces a network from that particular thresholded image CH(A).

In equations (6) and (7), A represents the thresholded image of the DEM and B the structuring template. The subscript n, ranging between 0 and N, is the size of the structuring template. In equation (6), the opening operation of the eroded set is always by means of a structuring template of an arbitrary size. If the size of B increases correspondingly with that of the structuring template that is used to erode the thresholded region A, as stated in equation (8), the widths of drainage segments of a particular order will increase with the increasing order of the drainage segments.

$$\mathrm{CH}_n(A) = \frac{A \ominus B_n}{[(A \ominus B_n) \ominus B_n] \oplus B_n} \quad n = 0, 1, 2, \ldots, N \tag{8}$$

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \quad \oplus \quad \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \quad = \quad \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$$

Structuring template (B_1) Structuring template (B_1) Structuring template (B_2)

Figure 2. Square type of structuring template of different sizes.

From equations (6) and (7), the following equation is proposed to extract the drainage network from the digital elevation model.

$$\mathrm{CH}(I) = \cup_{k=1}^{i-1} \frac{\cup_{n=0}^{N} \mathrm{CH}_n(A_k)}{A_{k+1}} \quad n = 0, 1, 2, \ldots, N \quad \forall k\,[0, i-1] \tag{9}$$

In the above equation logical difference is used for obvious reasons.

This algorithm extracts all the angular points or V-shaped crenulations. Equations (1) and (2) describe the threshold decomposition of the elevation regions represented in the grey levels. These thresholded binary images take the value 1 or 0. The remaining part of the algorithm tries to isolate the possible crenulations that are present in the successive elevation contours. Equations (3), (4) and (5) represent the fundamental morphological transformations to be performed on the binary images. Equations (6), (7), (8) and (9) are the sequential phases to extract the channel network from the DEM. Equation (6) isolates the V-shaped crenulations in all directions. The eroded image of a particular degree is opened by an arbitrary structuring template, and this opened image is subtracted from the eroded image to isolate a particular order of the crenulation. The degree of erosion decides the order of the crenulations. All the thresholded images will be further decomposed into crenulations. The union of crenulations of all thresholded images shown in the form of equation (9) will give the channel network path of the DEM considered.

3. Sample study

Of late, digital elevation models (DEM) are being consistently used in most of the studies related to watersheds. The DEMs derived directly through automated DEM generation that take advantage of stereo viewing capability of satellite images are preferred to those derived indirectly from digitized topographic maps. These automated maps are comparable to, or better than, those obtained through topographic maps in rugged or mountainous regions (Franklin 1990). The present case study illustrates a simple and elegant methodology for extracting drainage networks from DEMs, in general, by successfully implementing it on a transcendentally generated elevation model (figure 3) obtained by considering a 3rd order Koch-Quadric fractal (fractal basin). This binary fractal basin is decomposed into various topologically prominent regions that are assumed to represent the characteristics such as elevation regions in grey levels and crenulations as in the case of natural DEMs. The darker the pixel in this DEM, the higher its elevation. This DEM shown in figure 3 is used to illustrate the methodology. The reasons for using the transcendentally generated DEM are twofold:

1. To show that the extracted channel network is an intricate pattern.
2. The unavailability of high resolution DEM data with all crenulations in the spatially distributed elevation regions.

This digital elevation model is transformed into a one pixel wide channel network as in the procedure detailed in the previous section. The DEM (figure 3) could be decomposed into 15 binary images. The sequence of all possible thresholded images is not shown. These thresholded images of the DEM are decomposed into their drainage subsets by following equations (6) and (7). Equation (8) may be followed to extract the drainage network such that the generated channel network possesses divergent widths with the change in bifurcation generation levels. However, equation (9) has been used to represent the channel network of the DEM. Applying this

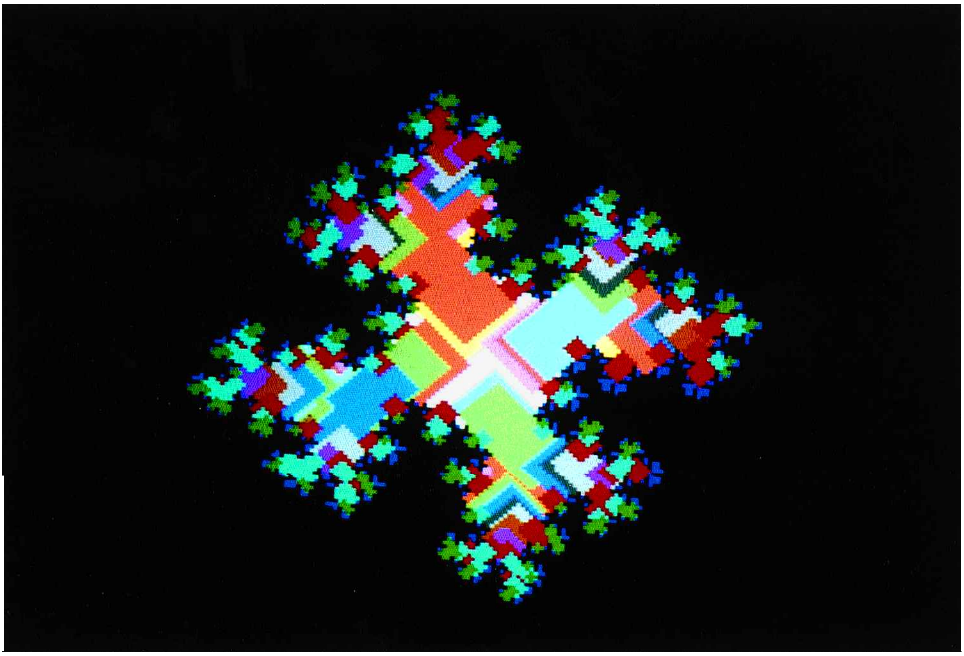

Figure 3. Simulated digital elevation model (DEM). A third order fractal koch-quadric is decomposed into topologically prominent regions. This is assumed to be a DEM. Light and dark regions are assumed to be spatially distributed low and high altitudes.

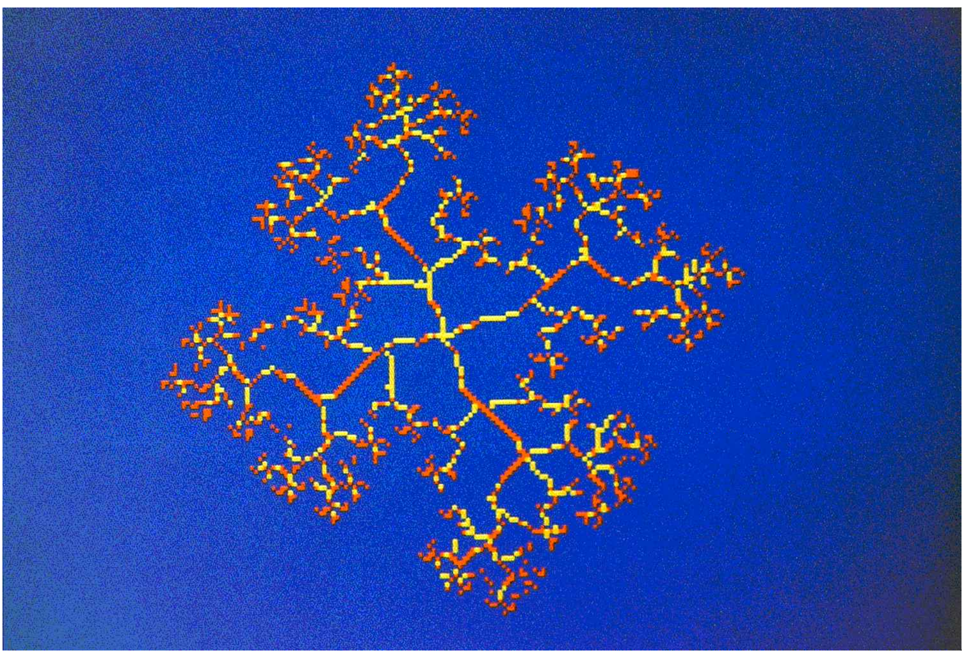

Figure 4. Drainage network in the simulated DEM.

procedure, all the thresholded images of the transcendentally generated DEM are decomposed into drainage subsets. The union of the channel subsets thus decomposed from the respective thresholded images produces the channel network of the DEM. Implementing the present channel network algorithm directly to the model leads to figure 4.

This study makes it amply clear that all real world drainage network extractions based on this methodology would definitely yield accurate and consistent results through the use of a real DEM generated through stereoscopic satellite imagery.

4. Conclusion

The morphological operators such as erosion, dilation, and cascade of erosion-dilation besides set subtraction and union operations have been methodically used in the present investigation. This algorithm is both computationally and algorithmically simple. The adaptation of this approach is straightforward without any limitations and it can be easily applied to extract channel networks from the natural DEMs. The sample study that is described here illustrates the interest of the morphological transformations. The present procedure should allow the use of morphological operators to extract channel networks. In particular, more experiments are presently being carried out to assess the completeness of these transformations for comprehending the channelization processes.

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