

Ground Rules: Time allowed is 10 minutes, individual work only and closed book test.

Your name

Solution

Score :

1. A lottery is held every day, and on any given day there is a 40% chance that someone will win, with each day independent of every other. Let Y denote the random variable describing the number of times in the next five days that the lottery will be won. Find $E[Y]$.

$$Y \sim \text{Binomial}(5, 0.4)$$

$$P(Y=k) = \binom{5}{k} (0.4)^k (0.6)^{5-k} \quad k=0, 1, 2, 3, 4, 5$$

$$E[Y] = \sum_{k=0}^5 k \cdot \binom{5}{k} (0.4)^k (0.6)^{5-k}$$

$$= \sum_{k=1}^5 \frac{5!}{(k-1)!(5-k)!} (0.4)^k (0.6)^{5-k}$$

$$= 5 \binom{4}{k-1} (0.4)^k (0.6)^{5-k}$$

$$= 5 (0.4) \sum_{m=0}^4 \frac{4!}{m!(4-m)!} (0.4)^m (0.6)^{4-m}$$

$$= 5 (0.4) (0.4 + 0.6)^4 = 5 \cdot 0.4 \cdot 1^4 = 2$$

□