Your name ___Solution___       Score : ☐

1. A lottery is held every day, and on any given day there is a 40% chance that someone will win, with each day independent of every other. Let $Y$ denote the random variable describing the number of times in the next five days that the lottery will be won. Find $E[Y]$.

$$Y \sim \text{Binomial}(5, 0.4)$$

$$P(Y=k) = {}^5C_k \cdot (0.4)^k (0.6)^{5-k} \qquad k=0,1,2,3,4,5$$

$$E[Y] = \sum_{k=0}^{5} k \cdot {}^5C_k (0.4)^k (0.6)^{5-k}$$

$$= \sum_{k=1}^{5} \frac{5!}{(k-1)!\,(5-k)!} (0.4)^k (0.6)^{5-k}$$

$$= 5\,(0.4) \cdot \sum_{m=0}^{4} \frac{4!}{m!\,(4-m)!} (0.4)^m (0.6)^{4-m}$$

$$= 5 \cdot (0.4)\,(0.4+0.6)^4 = 2$$

$\square$