

Ground Rules: Time allowed is 10 minutes, individual work only and closed book test.

Your name Solution

Score :

1. An urn has four balls labeled 1, 2, 3, and 4. A first ball is drawn and its number is denoted by X . A second ball is then drawn from the three remaining balls in the urn and its number is denoted by Y . Calculate $P(Y=3|X=2)$ and decide if X and Y are independent.

$$P(Y=3|X=2) = \frac{P(Y=3, X=2)}{P(X=2)} \quad - (*)$$

$$P(X=2) = P(\text{first ball is: } 2) = \frac{1}{4}$$

$$P(Y=3|X=2) = \frac{1}{3}$$

$$\begin{aligned} P(Y=3) &= P(Y=3, X=1) + P(Y=3, X=2) + P(Y=3, X=4) \\ &= P(Y=3|X=1) \cdot P(X=1) + P(Y=3|X=2) \cdot P(X=2) \\ &\quad + P(Y=3|X=4) \cdot P(X=4) \\ &= 3 \left(\frac{1}{3} \cdot \frac{1}{4} \right) = \frac{1}{4} \end{aligned}$$

$(*) \Rightarrow$

$$P(Y=3, X=2) = P(Y=3|X=2) \cdot P(X=2) = \frac{1}{12}$$

$$P(X=2) \cdot P(Y=3) = \frac{1}{16}$$

$\Rightarrow X$ & Y are NOT independent.