

Ground Rules: Time allowed is 20 minutes, individual work only and closed book test.

Your name _____

Score :

1. The heights of women are normally distributed with a mean of 65 inches and a standard deviation of 2.5 inches. My daughter is 63.35 inches. Using the normal tables (backside), find the percentage of women that are taller than my daughter.

Let X be the random variable which represents the heights of women.

Given $X \sim \text{Normal}(\mu, \sigma^2)$ where $\mu = 65$

$$\sigma = 2.5$$

We want to calculate

$$\mathbb{P}[X > 63.35]$$

Define $Z = \frac{X - \mu}{\sigma}$. Since $X \sim \text{Normal}(\mu, \sigma^2)$, So

Z is Normal(0, 1)

$$\text{Now } \mathbb{P}[X > 63.35] = \mathbb{P}[Z > \frac{63.35 - 65}{2.5}] = \mathbb{P}[Z > -0.66]$$

Since $-Z$ is also Normal(0, 1) } Normal Random Variables are symmetric about mean

$$\text{we get } \mathbb{P}[Z > -0.66] = \mathbb{P}[Z < 0.66] = \mathbb{P}[Z \leq 0] + \mathbb{P}[0 < Z < 0.66]$$

Again since Normal Variables are symmetric about mean,

$$\mathbb{P}[Z < 0] = \frac{1}{2}$$

$$\text{And } \mathbb{P}[0 < Z < 0.66] = \frac{1}{\sqrt{2\pi}} \int_0^{0.66} e^{-x^2/2} dx = 0.2454 \quad \left\{ \begin{array}{l} \text{From} \\ \text{table} \end{array} \right.$$

$$\text{So } \mathbb{P}[X > 63.35] = 0.5 + 0.2454 = 0.7454$$

So, percentage of women taller than given person is

$$74.54\%$$