

Theorem 1.3.8 $n \geq 2$, $\{A_i\}_{i=1}^n$ be a collection of events

Assume $P(\bigwedge_{i=1}^n A_i) > 0$

$$\text{Then } P(\bigwedge_{i=1}^n A_i) = P(A_n | \bigwedge_{i=1}^{n-1} A_i) P(\bigwedge_{i=1}^{n-1} A_i)$$

$$P(\bigwedge_{i=1}^n A_i) = P(A_1) \prod_{j=2}^n P(A_j | \bigwedge_{k=1}^{j-1} A_k)$$

Proof:- Ex. - use induction

Example 1.3.6 (Review)

	B1	B2	B3
Pink	4	3	3
Green	3	3	4
blue	5	2	3

- A Green ball is chosen. What is probability box 3 was chosen?

$G = \{\text{Green ball was chosen}\}$ $B_3 = \{\text{Box 3 was chosen}\}$

$$P(G|B_3) = \frac{4}{10}$$

$$Q: P(B_3|G) = ?$$

$$P(B_3|G)$$

$$= \frac{P(B_3 \cap G)}{P(G)}$$

Thm 1.3.5

$$B_i \cap B_j = \emptyset \quad (i \neq j)$$

$$S = \bigcup_{i=1}^3 B_i$$

$$P(G|B_3) \cdot P(B_3)$$

$$\sum_{i=1}^3 P(G|B_i) P(B_i)$$

$$= \frac{4/10 \cdot \frac{1}{3}}$$

$$\frac{3}{12} \cdot \frac{1}{3} + \frac{3}{8} \cdot \frac{1}{3} + \frac{4}{10} \cdot \frac{1}{3}$$

3 boxes

(Bayes) Theorem 1.3.11

Suppose we have an event A , $P(A) > 0$. Let $\{B_i : 1 \leq i \leq n\}$ be a collection of disjoint events, with $P(B_i) > 0$ $1 \leq i \leq n$.

Suppose $A \subseteq \bigcup_{i=1}^n B_i$ then $P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^n P(A|B_j)P(B_j)} \quad 1 \leq i \leq n$

Example 1-3-12

residents.

positive.

It is found that

Shyam is randomly selected from ISI-B

His blood is tested for Dengue & the result

95% of people with Dengue test

2% of people without Dengue test

1% of residents have Dengue

Q.6x What is Probability that Shyam has Dengue?

$S = \{ \text{shyam has Dengue} \}$

$D = \{ \text{shyam tests positive} \}$

$$P(D|S) = 0.95$$

$$P(D|S^c) = 0.02$$

$$P(S) = 0.01, P(S^c) = 0.99$$

$$Q: P(S|D)?$$

Bayes Theorem

$n=2$

$B_1 = S, B_2 = S^c$

$A = D$

$$P(S|D)$$

$$= \frac{P(S \cap D)}{P(D)}$$

$$= \frac{P(D|S) P(S)}{P(D|S) P(S) + P(D|S^c) P(S^c)}$$

$$= \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.02)(0.99)}$$

$$= 0.324$$

$$\square$$

1.4 Independence

We have seen how the occurrence of one event affects the probability of another event. It also happens that one event has no effect on another event.

We toss a coin 3 times

$\boxed{\text{Uniform}(S)} \equiv \begin{pmatrix} \text{Equally likely} \\ \text{outcome} \end{pmatrix}$

$$S = \{ hhh, hht, hth, thh, htt, tht, tth, ttt \}, \mathbb{P} = \mathcal{P}(S), \mathbb{P}$$

$$\text{st toss is a head} = \{ hhh, hht, hth, htt \}$$

$$\text{toss is a head} = \{ hhh, hht, thh, tht \}$$

$$A \cap B = \{ hhh, hht \}$$

$$P(A \cap B) = \frac{1}{4}$$

$$A \cap B^c = \{ hth, htt \} \quad P(A \cap B^c) = \frac{1}{4}$$

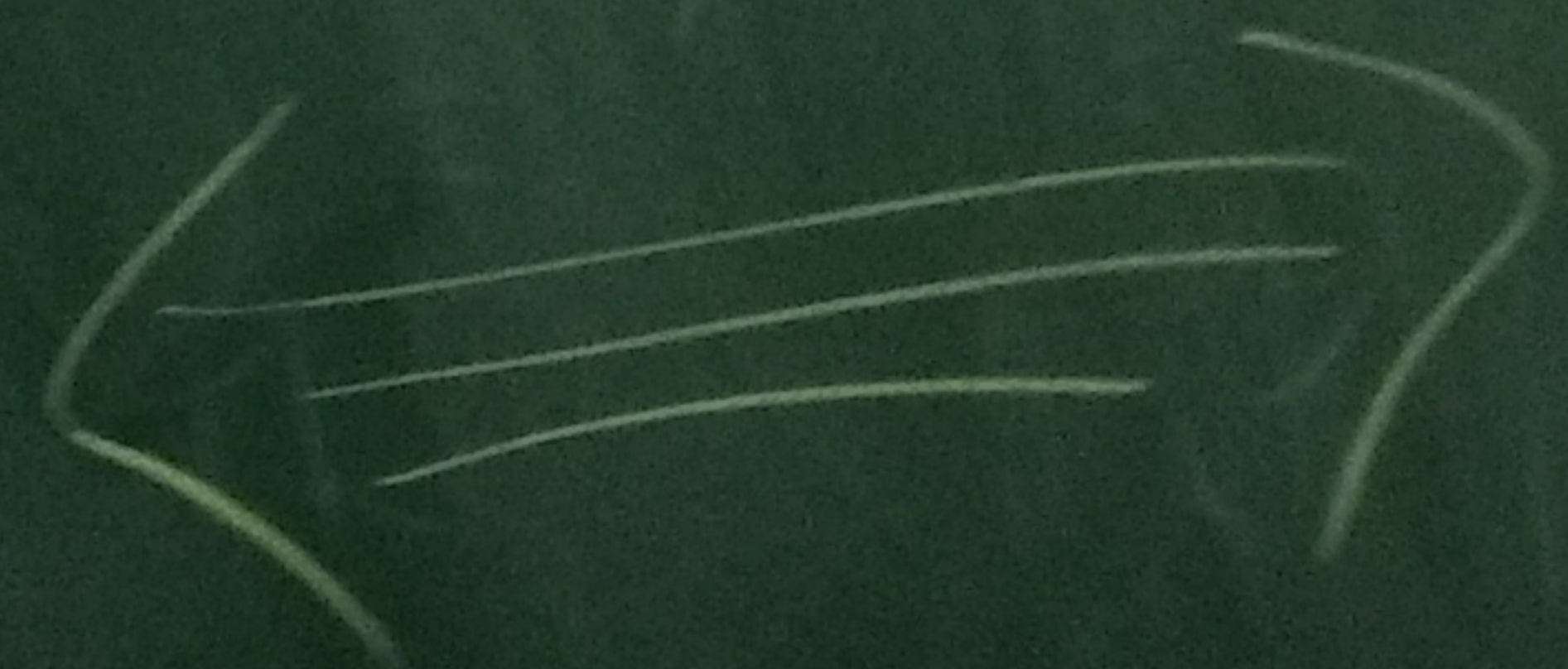
Observe

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{1}{2}$$

$$P(A|B) = P(A) \quad ?$$

requires $P(B) > 0$



$$P(A \cap B) = P(A)P(B) \quad "$$

1.4 Independence
Definition 1.4.2 (Independence)

Two events A & B are independent if

$$P(A \cap B) = P(A)P(B) \quad (*)$$

Remark: $(*)$ is 4 equations in 1.

$$\begin{aligned} & 0 < P(B) \\ & 0 < P(A) \end{aligned}$$

$$(*) \Rightarrow \begin{cases} P(A) = P(A|B) = P(A|B^c) \\ P(B) = P(B|A) = P(B|A^c) \end{cases}$$

(*) $\Rightarrow$

$$\boxed{P(A \cap B) = P(A)P(B)}$$

$$= P(A)(1 - P(B^c))$$

$$= P(A) - P(B^c)P(A)$$

$\Rightarrow$

$$P(A \cap B) - P(A) = -P(B^c)P(A)$$

$$\Rightarrow P(A) - P(A \cap B) = P(A)P(B^c)$$

(Thm 1.45
vi)

$$\Rightarrow \boxed{P(A \cap B^c) = P(A)P(B^c)}$$

$\Rightarrow$
 E_x

$$\boxed{P(A^c \cap B) = P(A^c)P(B)}$$

$$\boxed{P(A^c \cap B^c) = P(A^c)P(B^c)}$$

$\Uparrow$ E_x