Due: Tuesday, September 29th, 2016
Problem to be turned in: 2,6

1. A lottery is held every day, and on any given day there is a 30% chance that someone will win, with each day independent of every other. Let X denote the random variable describing the number of times in the next five days that the lottery will be won.
   (a) What type of random variable (with what parameter) is X?
   (b) On average (expected value), how many times in the next five days will the lottery be won?
   (c) When the lottery occurs for each of the next five days, what is the most likely number (mode) of days there will be a winner?
   (d) How likely is it the lottery will be won in either one or two of the next five days?

2. A game show contestant is asked a series of questions. She has a probability of 0.88 of knowing the answer to any given question, independently of every other. Let Y denote the random variable describing the number of questions asked until the contestant does not know the correct answer.
   (a) What type of random variable (with what parameter) is Y?
   (b) On average (expected value), how many questions will be asked until the first question for which the contestant does not know the answer?
   (c) What is the most likely number of questions (mode) that will be asked until the contestant does not know a correct answer?
   (d) If the contestant is able to answer twelve questions in a row, she will win the grand prize. How likely is it that she will know the answers to all twelve questions?

3. Sonia sends out invitations to eleven of her friends to join her on a hike she's planning. She knows that each of her friends has a 59% chance of deciding to join her independently of each other. Let Z denote the number of friends who join her on the hike.
   (a) What type of random variable (with what parameter) is Z?
   (b) What is the average (expected value) number of her friends that will join her on the hike?
   (c) What is the most likely number (mode) of her friends that will join her on the hike?
   (d) How do your answers to (b) and (c) change if each friend has only a 41% chance of joining her?

4. A player rolls three dice and earns \$1 for each die that shows a 6. How much should the player pay to make this a fair game?

5. **("The St.Petersburg Paradox")** Suppose a game is played whereby a player begins flipping a fair coin and continues flipping it until it comes up heads. At that time the player wins a 2^n dollars where n is the total number of times he flipped the coin. Show that there is no amount of money the player could pay to make this a fair game.

6. A random variable X has a probability mass function given by

$$P(X = 0) = 0.2, P(X = 1) = 0.5, P(X = 2) = 0.2, \text{ and } P(X = 3) = 0.1.$$

   Calculate the expected value and standard deviation of this random variable. What is the probability this random variable will produce a result more than one standard deviation from its expected value?

7. Answer the following questions about flips of a fair coin.

   (a) Calculate the standard deviation of the number of heads that show up in 100 flips of a fair coin.

   (b) Show that if the number of coins is quadrupled (to 400) the standard deviation only doubles.

8. Suppose we begin rolling a die, and let X be the number of rolls needed before we see the first 3.

   (a) Show that $E[X] = 6$.

   (b) Calculate $SD[X]$.

   (c) Viewing $SD[X]$ as a typical distance of X from its expected value, would it seem unusual to roll the die more than nine times before seeing a 3?

   (d) Calculate the actual probability $P(X > 9)$.

   (e) Calculate the probability X produces a result within one standard deviation of its expected value.