Due: Tuesday, September 6th, 2016
Problem to be turned in: 2,6

1. Let X and Y be random variables with joint distribution given by the chart below.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	1/12	0	3/12
$Y = 1$	2/12	1/12	0
$Y = 2$	3/12	1/12	1/12

   (a) Compute the marginal distributions of X and Y.
   (b) Compute the conditional distribution of X given that $Y = 2$.
   (c) Compute the conditional distribution of Y given that $X = 2$.
   (d) Carry out a computation to show that X and Y are not independent.

2. Consider six independent trials each of which are equally likely to produce a result of 1, 2, or 3. Let X_j denote the number of trials that result in j. Calculate $P(X_1 = 1, X_2 = 2, X_3 = 3)$.

3. Let $X \sim \text{Uniform}(\{1, 2, 3\})$ and $Y \sim \text{Uniform}(\{1, 2, 3\})$ be independent and let $Z = X + Y$.
   (a) Determine the range of Z.
   (b) Determine the distriubtion of Z.
   (c) Is Z uniformly distributed over its range?

4. Consider the experiment of rolling three dice and calculating the sum of the rolls. Answer the following questions.
   (a) What is the range of possible results of this experiment?
   (b) Calculate the probability the sum equals three.
   (c) Calculate the probability the sum equals four.
   (d) Calculate the probability the sum equals five.
   (e) Calculate the probability the sum equals ten.

5. Let $X \sim \text{Binomial}(n, p)$ and $Y \sim \text{Binomial}(m, p)$. Assume X and Y are independent and let $Z = X + Y$. Prove that $Z \sim \text{Binomial}(m + n, p)$.

6. Let $X \sim \text{Negative Binomial}(r, p)$ and $Y \sim \text{Negative Binomial}(s, p)$. Assume X and Y are independent and let $Z = X + Y$. Prove that $Z \sim \text{Negative Binomial}(r + s, p)$.

7. Consider one flip of a single fair coin. Let X denote the number of heads on the flip and let Y denote the number of tails on the flip.
   (a) Show that $X, Y \sim Bernoulli(\frac{1}{2})$.
   (b) Let $Z = X + Y$ and explain why $P(Z = 1) = 1$.
   (c) Since (b) clearly says that Z cannot be a $Binomial(2, \frac{1}{2})$, explain why this is the case.

8. Let $X \sim \text{Geometric}(p)$ and $Y \sim \text{Geometric}(p)$ be independent and let $Z = X + Y$.
   (a) Determine the range of Z.
   (b) Use a convolution to prove that $P(Z = n) = (n - 1)p^2(1 - p)^{n-2}$.
   (c) Recall from the discussion of Geometric distributions that $(X = 1)$ is the most likely result for X and $(Y = 1)$ is the most likely result for Y. This does not imply that $(Z = 2)$ is the most likely outcome for Z. Determine the values of p for which $P(Z = 3)$ is larger than $P(Z = 2)$.