

Due: Tuesday, August 30th, 2016**Due: Thursday, September 1st, 2016***Problem to be turned in: 2**Problem to be turned in: 7*

1. Consider the experiment of flipping a coin four times and recording the sequence of heads and tails. Let S be the sample space of all sixteen possible orderings of the results. Let X be the function on S describing the number of tails among the flips. Let Y be the function on S describing the first flip (if any) to come up tails.
 - (a) Create a table as in Example 3.1.1. describing functions X and Y .
 - (b) Use the table to calculate $P(X = 2)$.
 - (c) Use the table to calculate $P(Y = 3)$.
2. A pair of fair dice are thrown. Let X represent the larger of the two values on the dice and let Y represent the smaller of the two values.
 - (a) Describe S , the domain of functions X and Y . How many elements are in S ?
 - (b) What are the ranges of X and Y . Do X and Y have the same range? Why or why not?
 - (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that X and Y have the same distribution ?
3. A pair of fair dice are thrown. Let X represent the number of the first die and let Y represent the number of the second die.
 - (a) Describe S , the domain of functions X and Y . How many elements are in S ?
 - (b) Describe T , the range of functions X and Y . How many elements are in T ?
 - (c) Describe the distribution of X and describe the distribution of Y by finding the probability mass function of each. Is it true that X and Y have the same distribution ?
 - (d) Are X and Y the same function? Why or why not?
4. Use the \sim notation to classify the distributions of the random variables described by the scenarios below. For instance, if a scenario said, “let X be the number of heads in three flips of a coin” the appropriate answer would be $X \sim \text{Binomial}(3, \frac{1}{2})$ since that describes the number of successes in three Bernoulli trials.

- (a) Let X be the number of 5's seen in four die rolls. What is the distribution of X ?
- (b) Each ticket in a certain lottery has a 20% chance to be a prize-winning ticket. Let Y be the number of tickets that need to be purchased before seeing the first prize-winner. What is the distribution of Y ?
- (c) A class of ten students is comprised of seven women and three men. Four students are randomly selected from the class. Let Z denote the number of men among the four randomly selected students. What is the distribution of Z ?
5. Let $X : S \rightarrow T$ be a discrete random variable. Suppose $\{B_i\}_{i \geq 1}$ are sequence of events in T then show that $X^{-1}(\bigcup_{i=1}^{\infty} B_i) = \bigcup_{i=1}^{\infty} X^{-1}(B_i)$ and that if B_i and B_j are disjoint, then so are $X^{-1}(B_i)$ and $X^{-1}(B_j)$.
6. An urn has four balls labeled 1, 2, 3, and 4. A first ball is drawn and its number is denoted by X . A second ball is then drawn from the three remaining balls in the urn and its number is denoted by Y .
- (a) Calculate $P(X = 1)$.
- (b) Calculate $P(Y = 2|X = 1)$.
- (c) Calculate $P(Y = 2)$.
- (d) Calculate $P(X = 1, Y = 2)$.
- (e) Are X and Y independent? Why or why not?
7. Two dice are rolled. Let X denote the sum of the dice and let Y denote the value of the first die.
- (a) Calculate $P(X = 7)$ and $P(Y = 4)$.
- (b) Calculate $P(X = 7, Y = 4)$.
- (c) Calculate $P(X = 5)$ and $P(Y = 4)$.
- (d) Calculate $P(X = 5, Y = 4)$.
- (e) Are X and Y independent? Why or why not?