Due: Tuesday, August 16th, 2016
Problems to be turned in: 5,6

Due: Thursday, August 18th, 2016
Problems to be turned in: 9,11

1. A manufacturer produces nuts and markets them as having 50mm radius. The machines that produce the nuts are not perfect. From repeated testing, it was established that 15% of the nuts have radius below 49mm and 12% have radius above 51mm. If two nuts are randomly (and independently) selected, find the probabilities of the following events:
   (a) The radii of both the nuts are between 49mm and 51mm;
   (b) The radius of at least one nut exceeds 51mm.

2. Four tennis players (Avinash, Ben, Carlos, and David) play a single-elimination tournament with Avinash playing David and Ben playing Carlos in the first round and the winner of each of those contests playing each other in the tournament final. Below is the chart giving the percentage chance that one player will beat the other if they play. For instance, Avinash has a 30% chance of beating Ben if they happen to play.

	Avinash	Ben	Carlos	David
Avinash	-	30%	55%	40%
Ben	-	-	80%	45%
Carlos	-	-	-	15%
David	-	-	-	-

   Suppose the outcomes of the games are independent. For each of the four players, determine the probability that player wins the tournament. Verify that the calculated probabilities sum to 1.

3. Let A and B be events with $P(A) = 0.8$ and $P(B) = 0.7$.
   (a) What is the largest possible value of $P(A \cap B)$?
   (b) What is the smallest possible value of $P(A \cap B)$?
   (c) What is the value of $P(A \cap B)$ if A and B are independent?

4. Suppose we toss two fair dice. Let E_1 denote the event that the sum of the dice is six. E_2 denote the event that sum of the dice equals seven. Let F denote the event that the first die equals four. Is E_1 independent of F? Is E_2 independent of F?

5. Suppose a bowl has twenty-seven balls. One ball is black, two are white, and eight each are green, red, and blue. A single ball is drawn from the bowl and its color is recorded. Define

$$\begin{aligned} A &= \{\text{the ball is either black or green}\} \\ B &= \{\text{the ball is either black or red}\} \\ C &= \{\text{the ball is either black or blue}\} \end{aligned}$$

   (a) Calculate $P(A \cap B \cap C)$.
   (b) Calculate $P(A)P(B)P(C)$.
   (c) Are A, B, and C mutually independent? Why or why not?

6. When can an event be independent of itself? Do parts (a) and (b) below to answer this question.
   (a) Prove that if an event A is independent of itself then either $P(A) = 0$ or $P(A) = 1$.

(b) Prove that if A is an event such that either $P(A) = 0$ or $P(A) = 1$ then A is independent of itself.

7. Suppose that airplane engines operate independently in flight and fail with probability p $(0 \leq p \leq 1)$. A plane makes a safe flight if at least half of its engines are running. Kingfisher Air lines has a four–engine plane and Paramount Airlines has a two–engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight?

8. It is estimated that 0.8% of a large shipment of eggs to a certain supermarket are cracked. The eggs are packaged in cartons, each with a dozen eggs, with the cracked eggs being randomly distributed. A restaurant owner buys 10 cartons from the supermarket. Call a carton "defective" if it contains at least one cracked egg.

   (a) If she notes the number of defective cartons, what are the possible outcomes for this experiment?

   (b) If she notes the total number of cracked eggs, what are the possible outcomes for this experiment?

   (c) How likely is it that she will find exactly one cracked egg among all of her cartons?

   (d) How likely is it that she will find exactly one defective carton?

   (e) Explain why your answer to (d) is close to, but slightly larger than, than your answer to (c).

   (g) What is the most likely number of cracked eggs she will find among her cartons?

   (h) What is the most likely number of defective cartons she will find?

   (i) How do you reconcile your answers to parts (g) and (h)?

9. A fair die is rolled repeatedly.

   (a) What is the probability that the first 6 appears on the fifth roll?

   (b) What is the probability that no 6's appear in the first four rolls?

   (c) What is the probability that the second 6 appears on the fifth roll?

10. For the problems below, assume the probability space is a Geometric(p) distribution with $0 < p < 1$. Show that the mode of a Geometric(p) distribution is 1.

11. Scott is playing a game where he rolls a standard die until it shows a 6. The number of rolls needed therefore has a Geometric($\frac{1}{6}$) distribution. Use the appropriate R commands to do the following:

   (a) Produce a vector of values for $j = 1, \ldots, 6$ corresponding to the probabilities that it will take Scott j rolls before he observes a 6.

   (b) Scott figures that since each roll has a $\frac{1}{6}$ probability of producing a 6, he's bound to get that result at some point after six rolls. Use the results from part (a) to determine the probability that Scott's expectations are met and a 6 will show up in one his first six rolls.

12. Suppose a fair coin is tossed n times. Compute the following:

   (a) $P(\{4 \text{ heads occur }\}|\{3 \text{ or } 4 \text{ heads occur}\})$;

   (b) $P(\{k-1 \text{ heads occur}\}|\{k-1 \text{ or } k \text{ heads occur}\})$; and

   (c) $P(\{k \text{ heads occur}\}|\{k-1 \text{ or } k \text{ heads occur}\})$.

13. Two coins are sitting on a table. One is fair and the other is weighted so that it always comes up heads.

   (a) If one coin is selected at random (each equally likely) and flipped, what is the probability the result is heads?

   (b) One coin is selected at random (each equally likely) and flipped five times. Each flip shows heads. Given this information about the coin flip results, what is the conditional probability that the selected coin was the fair one?