Due: April, 17th 2003.

1. The prices of a certain security follow a geometric Brownian motion with parameters $\mu = .12$ and $\sigma = 0.24$. If the security's price is presently 40 what is the probability that a call option, having four months until its expiration time and with a strike price of $K = 42$, will be exercised ?

2. If the interest rate is 8%, what is the risk neutral valuation of the call option specified in the previous problem.

3. What is the risk-neutral valuation of a six-month European put option to sell a security for a price 100 when the current price is 105, the interest rate is 10%, and the volatility of the security is .30 ?

4. A security's price follows a geometric Brownian motion with drift parameter 0.06 and volatility parameter 0.3.

   (a) What is the probability that the price of the security in six months is less than 90% of what it is today ?

   (b) Consider a newly instituted investment fitted, for an initial cost of A, returns you 100 in six months if the price at that times is less than 90% of what it intially was but returns you 0 otherwise. What must be the value of A in order for this investment's introduction not to allow an arbitrage ?

5. Th price of certain security follows a geometric Brownian motion with drift parameter $\mu = 0.05$ and volatility parameter $\sigma = 0.3$. The present price of the security is 95.

   (a) If the interest rate is 4%, find the no-arbitrage cost of a call option that expires in three months and has exercise price 100.

   (b) What is the probability that the call option in part (a) is worthless at the time of expiration ?

   (c) Suppose that a new type of investment on the security is being traded. This investment returns 50 at the end of one year if the price six months after purchasing the investment is at least 105 and the price one year after the purchase is at least as much as the price was after six months. Determine the no-arbitrage cost of this investment.

6. A European asset or nothing call pays its holder a fixed amount F if the price at expiration time if the price t expiration time is larger than K and pays 0 otherwise. Find the risk-neutral valuation of such a call - one that expires in six month's time and has $F = 100$and $K = 40$ -f if the present price security is 38, its volatility is 0.32 and the interest rate is 6%.

7. If the drift parameter of the geometric Brownian motion is 0, find the expected payoff of the asset-or-nothing call in the previous problem.

8. To determine the probability that a European call option finishes in the money, is it enough to specify the five parameters $K, S(0), r, t, \sigma$? Explain your answer, if it is "no", what else is needed ?

9. What should be the cost of a call option if the strike price is equal to zero ?

10. What should the cost of a call option become as the exercise time becomes larger and larger ? Explain.

11. What should the cost of a call option with Strike price=K and expiration time T become as the volatility becomes smaller ?