Due date: 28th, October 2010

1. Consider the ODE:
$$(1+t^2)\frac{d^2x}{dt^2} + 2t\frac{dx}{dt} - 2x = 0.$$
Find the solution in terms of power series in t. Find its radius of convergence.

2. Consider the ODE:
$$\frac{d^2x}{dt^2} + t\frac{dx}{dt} + x = 0.$$
Find the power series series solutions of the above and the corresponding radius of convergence.

3. Classify the singularities and find the roots of the indicial equation of :
$$t^3\frac{d^2x}{dt^2} + (\cos(2t) - 1)\frac{dx}{dt} + 2tx = 0$$

4. Classify the singularities and find the Frobenius solution(s) of the following ODEs:
$$t^2\frac{d^2x}{dt^2} - 3t\frac{dx}{dt} + (4t+4)x = 0$$
$$t\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + tx = 0$$

5. Consider the Bessel's equation :
$$t^2\frac{d^2x}{dt^2} + t\frac{dx}{dt} + (t^2 - \frac{1}{4})x = 0$$
Show that the indicial equation has two roots that differ by one. Can you still find two Frobenius series solutions ?