Quiz 3 on February 14

1. Fixed point iteration method was discussed in class. Prove the Theorem stated in class for this method (modify the statement so that optimal assumptions can be used for the method). o
2. Manually convert the following numbers to base 2: 5, 21, 35, 64. Check your conversion with the built-in `dec2bin` function.
3. Convert the following numbers to floating point values with eight-bit mantissas: 0.4, 0.5, 1.5
4. OCTAVE has inbuilt variables called `realmax` and `realmin` denoting the largest and the smallest numbers it can store.
   (a) Check that 10*`realmax` generates an overflow while `realmax` + 1 does not. Explain.
   (b) Using OCTAVE command window find the largest n such that $n!$ exceeds `realmax`. Briefly describe how you found it. o
5. Find enough terms in the Taylor Series for the function $f(x) = x(1 - \ln(x))$ at $x_0 = 2$, so that the truncation error is fourth-order $O((x - x_0)^4))$. Now once you have done that, procduce the following figure using subplot: The figure needs to arrange 4 plots in a 2 by 2 grid. In the top left, plot the function and its first-order Taylor series. In the top right, plot the function and its second order Taylor series. In the bottom left and right plots, do the same with the third and fourth-order Taylor series respectively. Plot the function as a dashed line, and the Taylor series as a solid line on the x-axis from 0 to 5.
6. Using (OCTAVE and) Newton's method approximate to within 10^{-4}, the value of x_0 which is the point on the graph of $y = x^2$ that is closest to $(1, 0)$.
7. Using `fx3n` and `newton`, find the root of the equation $x - x^{\frac{1}{3}} - 2 = 0$ with an initial guess $x_0 = 3$, with x-tolerance and f-tolerance to be within 5×10^{-16}. Note down the number of iterations required for convergence and the value of the root.
8. Let $r = 1$ and $s = 0.25$. Using the inbuilt function `roots`, solve for h, where $h^3 - 3rh^2 + 4sr^3$.
9. Consider $f(x) = x^3 - 7$. Find a positive root of $f(x) = 0$, with the help of a calculator. Now
   (a) Write a m-file function `bisection` that takes in as input a(left end point), b (right end point), n the number of iterations and performs the Bisection method for the above function.
   (b) Starting with the interval $[1, 2]$ perform 3 iterations. Compare your answer to the calculator answer.

Summary

At the begining of this week you should be able to

1. List the digits used in base two arithmetic.
2. Give a simple explanation of the term “floating point number”.
3. Give a definition of “roundoff error”.
4. Sketch the floating point number line and label its major features. Explain the expression, “There are holes in the floating point number line”.
5. Explain why integer arithmetic is “exact”? and why floating point arithmetic is not “exact”.
6. Identify (at least) two important differences between symbolic and numeric computations. Give one example of cancellation error.
7. Give a simple explanation of overflow.
8. Give a simple expression that defines machine precision, ϵ_m. Name the built in variable in OCTAVE that holds the value of ϵ_m.
9. With the value of ϵ_m for computations in OCTAVE, write a simple, but carefully coded `if` statement that determines whether two scalar values are close enough to be considered equal.
10. Write the formulas for computing the relative and absolute errors if α is an exact (scalar) value, and $\hat{\alpha}$ is its floating point approximation.
11. Identify the truncation error of a Taylor series expansion. Use an infinite series to give an example of truncation error and use order notation to express it.
12. Be able to distinguish the effects of roundoff and truncation errors in a computed result,
13. Explain the role of bracketing. Write a simple equation that expresses the condition for finding a root in a bracket interval.
14. Manually perform a few steps of the bisection method. Identify the one situation where bisection will return an incorrect value for x as a root.
15. Manually perform a few steps of the Newton's method and secant method
16. Identify situations that cause Newton's method to fail
17. Describe the possible expressions for convergence criteria. Specify convergence tolerance for any function so that excessive (unnecessary) iterations of a root-finder are not performed.
18. Describe the procedure used by `roots` to find the roots of a polynomial.
19. Qualitatively compare the convergence rates of bisection, secant and Newton's method
20. To perform basic root-finding with OCTAVE you will need to
    (a) Plot any $f(x)$ as a means of graphically identifying the location of roots.
    (b) Write an m-file that evaluates $y = f(x)$ for use with `bisect`, and `secant`.
    (c) Write an m-file that evaluates $f(x)$ and $f'(x)$ for use with the `newton` function
    (d) Find zeros of a function with the `bisect` and `newton`,
    (e) Find roots of polynomials with the `roots` command.