Measure and Probability: (ISBN 81 7371 613 7) Typos list

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Errata List: If you find any errors (typographical or otherwise) and they are not on the list below then please email us at athreya@isibang.ac.in or sunder@imsc.res.in. We will really appreciate it. Thanks.

Page VII, Line 3 ".... Talent Search Programme' at the Fergusson College, Pune in 1993,..." should read ".... Talent Search Programme' at IIT, Mumbai in 1993,..."
Page VIII, 3rd para, line-3: "Bernouilli" should be "Bernoulli"
Page VIII, 4th para, line 7: "Caratheorodory" should be "Caratheodory"
Page 1, 2nd para: There should be a ")" after 'probability of event'.
Page 2, line -12: ":" should be replaced by "."
Page 6, line 11 "$\mathcal{A}, B, M, S$" should be "$\mathcal{A}, \mathcal{B}, \mathcal{M}, \mathcal{S}$"
Page 7, line 10 Replace

$$\begin{aligned}\mathcal{M} &= \{A \in \mathcal{M}(A) : \mathcal{M}_A = \mathcal{M}(A)\} \\ &= \{A \in \mathcal{M}(A) : E \in \mathcal{M}(A) \Rightarrow E \cup A \in \mathcal{M}(A)\}.\end{aligned}$$

by

$$\begin{aligned}\mathcal{M} &= \{A \in \mathcal{M}(\mathcal{A}) : \mathcal{M}_A = \mathcal{M}(\mathcal{A})\} \\ &= \{A \in \mathcal{M}(\mathcal{A}) : E \in \mathcal{M}(\mathcal{A}) \Rightarrow E \cup A \in \mathcal{M}(\mathcal{A})\}.\end{aligned}$$

Page 38, Ex 2.2.15: Last line should read as

$$\int_0^1 e^{-s} \log s ds = \lim_{n\to\infty} \int_{\frac{1}{n}}^1 (1 - \frac{s}{n})^n \log(s) ds$$

Page 60, Ex 4.1.10:should read as:

Ex 4.1.10: Let $\Omega_1 = \Omega_2 = \mathbb{N}$, $\mathcal{B}_1 = \mathcal{B}_2 = \mathcal{P}(\mathbb{N})$. Define the measures μ_i on $(\Omega_i, \mathcal{B}_i)$ by $\mu_i(\{k\}) = 2^{-k}$. Define $f : \Omega_1 \times \Omega_2 \to \mathbb{R}$ by

$$f(m,n) = \begin{cases} -n2^{2n} & \text{if } m = n \\ n2^{2n} & \text{if } m = n-1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that f is $\mathcal{B}_1 \otimes \mathcal{B}_2$ measurable.
(b) Observe that

$$\int_{\Omega_2}\int_{\Omega_1} f(m,n)d\mu_1(m)d\mu_2(n) \neq \int_{\Omega_1}\int_{\Omega_2} f(m,n)d\mu_2(n)d\mu_1(m),$$

thereby emphasising the importance of integrability in the hypotheses of Fubini's theorem.

Page 43, Line -3: "Let X, Y be two random variables ..." should be replaced by "Let X be a random variable ..."

Page 45, Definition 3.2.1: "$P(\cap_{i=1}^n A_i) = \prod_{i=1}^n P(A_i)$" should be replaced by

$$P(\cap_{i=1}^n A_i^{\epsilon_i}) = \prod_{i=1}^n P(A_i^{\epsilon_i}),$$

where $\epsilon_i \in \{0,1\}$, $A_i^0 := A_i$ and $A_i^1 := A'$.

Page 70, paragraph 3: In the proof of Kolmogorov's Consistency Theorem (Th. 4.4.4), the third paragraph on page 70 should be changed to read as follows: So, suppose $\varepsilon > 0$. Since, by assumption, $P|\mathcal{B}_{n]}$ is a probability measure, we can, by Proposition 4.4.2, find a compact set $C_n \subseteq E_n$ such that $P(A_n \setminus B_n) \leq \varepsilon/2^{n+1}$, where $B_n = \pi_{n]}^{-1}(C_n)$. Note that if $\widetilde{A}_n = \cap_{k=1}^n B_k$, then $\widetilde{A}_n \in \mathcal{B}_{n]}$ and in fact $\widetilde{A}_n = \pi_{n]}^{-1}(K_n)$ where $K_n = \{(x_1, \cdots, x_n) \in C_n : (x_1, \cdots, x_k) \in C_k \text{ for } 1 \leq k \leq n\}$ is a compact subset of $\mathbb{R}^n$. Now

$$\begin{aligned} P(A_n \setminus \widetilde{A}_n) &= P(A_n \setminus \cap_{k=1}^n B_k) \\ &\leq \sum_{k=1}^n P(A_n \setminus B_k) \\ &\leq \sum_{k=1}^n P(A_k \setminus B_k) \\ &< \varepsilon/2 \end{aligned}$$

and hence, $P(\widetilde{A}_n) \geq \varepsilon/2$ for all n, and hence each K_n is a non-empty compact set.

Page 75, (5.1.2): should read as :

$$f_\sigma(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\phi_X(t)e^{-\frac{\sigma^2 t^2}{2}}dt$$

Page 76, line 2:

$$= \int_{\mathbb{R}} e^{-iay}\phi_X(y)e^{-\frac{\sigma^2 y^2}{2}}dy$$

should be changed to

$$= \frac{1}{2\pi}\int_{\mathbb{R}} e^{-iay}\phi_X(y)e^{-\frac{\sigma^2 y^2}{2}}dy$$

Page 76, Lemma 5.1.7: $(\Omega, \mathcal{B})$ should be replaced by $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$
Page 76, line -1: "real-valued" should be replaced by "non-negative"
Page 83, line -6: "$\forall n \geq m$" should be changed to "$\forall n \geq m_1$"
Page 83, line -7: "$\forall n \geq m$" should be changed to "$\forall n \geq m_1$"
Page 85, line -9: "$1 \leq i \leq n$ " should be changed to "$1 \leq i \leq n-1$"
Page 93, line 11: "$A \in \mathcal{B}$" should be changed to "$A \in \mathcal{F}$"
Page 116, Line 1: "$f : \mathbb{N} \to \mathbb{R}$" should be changed to "$f : S \to \mathbb{R}$"
Page 140, (7.1.3): should read as

$$E_1, E_2 \in \mathcal{B},\ E_2 \in \mathcal{C}_F,\ E_1 \subset E_2\ \Rightarrow\ E_1 \in \mathcal{C}_F.$$

Page 140, line -6: ".. in the notation of (3).." should be changed to "..in the notation of (2).."
Page 150, line 4: "X" should be changed to "Ω".
Page 153, line -4: should be changed to

$$\Longrightarrow g1_E = 1_E\ \ \rho \text{ a.e.}$$

Page 160, line 8: should be changed to

$$|x - m_k| \leq \lambda(I) + \frac{1}{2}\lambda(I_k) \leq s_{k-1} + \frac{1}{2}\lambda(I_k) < 2\lambda(I_k) + \frac{1}{2}\lambda(I_k) = \frac{5}{2}\lambda(I_k).$$

Page 160, line -7: "$\mathbb{R}$" should be changed to "$[a, b]$" and "$x \in \mathbb{R}$" should be changed to "$x \in [a, b]$"
Page 161, line 17: "Proposition 4.4.1" should be changed to "Lemma

4.4.1"

Page 161, line -3: "such that $J_j^y = (y-j, y+j) \subset I_i$" should be replaced by "such that $J_j^y = (y-k_j, y+k_j) \subset I_i$"

Page 161, line -1:

$$f(y+j) - f(y) > qj,$$

should be changed to

$$f(y+j) - f(y) > qk_j,$$

Page 162, line 1: "for all $j \leq k(y)$." should be changed to "for all $k_j \leq k(y)$."

Page 162, line 7: "$= p \sum_{k=1}^n h_k$" should be changed to "$= 2p \sum_{k=1}^n h_k$ "

Page 163, line -1: "$N^a(x) := \sup_{\pi \in \mathcal{P}[a,x]} \sum_{i=1}^k -[(f(x_i) - f(x_{i-1})^-$" should be replaced by "$N^a(x) := \sup_{\pi \in \mathcal{P}[a,x]} \sum_{i=1}^k [(f(x_i) - f(x_{i-1})^-$"

Page 164, Proposition 7.5.7: Here and for the rest of the section: λ denotes Lebesgue measure.