INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
MS-QMS - First Year, First Semester, 2014-15
Probability-I, Semesteral Examination

1. Suppose a factory has two machines A and B that make 40% and 60% of the total production, respectively. 2% of A's output and 3% of B's output is defective.
(a) If a product is selected at random from the output what is the probability that it was produced by machine A?
(b) If a product is selected at random from the output what is the probability that it is defective?
(c) Given a defective product, what is the probability that it was produced by machine B? [24]

2. Suppose the joint probability density function of (X, Y) is given by

$$f(x,y) = \begin{cases} \lambda^2 \exp(-\lambda y) & \text{if } \infty > y > x > 0; \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal probability distributions of X and Y.
(b) Find $E(Y)$, $Var(Y)$ and $Cov(X, Y)$. [24]

3. (a) State the *Central Limit Theorem* applicable to a sequence of i.i.d. random variables with finite variance.
(b) Let $U_1, U_2, \ldots$ be an i.i.d. sequence of $U(1, 3)$ random variables and set $S_n = \sum_{i=1}^{n} U_i$. Find (approximately) $P(590 \leq S_{300} \leq 620)$. [20]

Cumulative Normal Probability Tables (Z-Values)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998

SQC & OR Unit

Indian statistical Institute,8th Mile Mysore Road,Bangalore-59

M.S.(Quality Management Science) (2014-2015)
Semester I-July 2014

Paper :Reliability, Maintainability and Safety

Time: 3 Hours Date: 10 November, 2014 Max. Marks: 100

PART – II (30 marks)

This part carries "40" Marks.Answer as many questions as you can but the maximum marks you can score is "30"

Question (1): Tick the most appropriate answer for the following questions with justification

a) The plot of a Probability Density Function(p.d.f) shows
   i. Failure rate versus time
   ii. A positive slope wherever failure rate is constant
   iii. Number of events(such as failures) versus time
   iv. Number of success versus total no. of opportunities (2)

b) An analysis of historic data indicates that the repair time for a particular product can be modeled by the Lognormal Distribution with $\mu=1.7$ hrs and $\sigma=0.65$ hrs.The estimate for MTTR (Mean Time to Repair) is

   i. 5.5 hrs
   ii. 6.8 hrs
   iii. 7.4 hrs
   iv. 8.1 hrs (2)

c) A company has warranted its monitors for 3000 hrs of service. The monitors have constant failure rate and have a MTTF of 20,000 hrs.About what % of the monitors are functioning at the end of the warranty period?
   i. 86%
   ii. 92%
   iii. 1.16%
   iv. 36.8%
   v. None of the above (2)

d) The system reliability of a parallel system
   i. Decreases as more redundant subsystems are added to the system
   ii. Increases if the subsystem with the lowest reliability is removed
   iii. Is greater than the reliability of any subsystem
   iv. Is equal to the reliability of the "best" subsystem (3)

e) The time to fail for a flexible membrane follows the two parameter Weibull distribution with $\beta = 2$ and $\eta = 300$ months. After how many months 90% reliability is achieved?
   i. 60.51 months
   ii. 97.38 months
   iii. 107.36 months
   iv. None of the above (4)

f) A system has three subsystems with a reliability of "R".System success requires that at least two of the subsystems operate. The system reliability can be calculated as:
   i. $3R^3 - 2R^2$
   ii. $2R^3 - 3R^2$
   iii. $3R^2 - 2R^3$
   iv. $2R^2 - 3R^3$ (3)

Question (2): Explain the following
(a) Failure Density Function f(t) versus The Hazard Function h(t)
(b) For a given hazard function h(t), the expression of Reliability Function R(t) in terms of Hazard Function h(t)

(5+5=10)

Question (3): The time between failures of a laser in a cytogenics machine is exponentially distributed with a mean of 25,000 hrs.
(a) What is the expected time until the second failure?
(b) What is the probability that the time until the third failure exceeds 50,000hrs? (2+5=7)

Question (4): Calculate the reliability for a 10 hour operating period of a (i) parallel system with two units and (ii) a two unit standby system using the same units. Each of the units has failure rate of 0.01 failure/hr and assume 100% reliability of sensing, switching and idling. Compare the two systems on the basis of reliability and MTBF. (7)