Paper 12: Elements of Maths 2 - End-Semester Exam

MS LIS First Year

May 02, 2017

Instructions: There are **2 questions** altogether. Answer as many questions as you can. Marks corresponding to each question is indicated in bold. Maximum score : **60 marks.** Maximum time : **3 hrs.**

(1) For each of the following questions, write the final answer (you need not show the calculations):

(a) Suppose $A = \{0, 1, 2, \cdots, 7\}$, $B = \{0, 1\}$ and $f : A \to B$ is given by $f(x) = x - |x|$. What is the range of f?

(b) If $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = x^3 - 1$. Find $f^{-1}(1)$?

(c) If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{x \to 2^-} f(x) = 10$ then find all the possible values of $f(2)$?

(d) If $f, g : \mathbb{R} \to \mathbb{R}$ are given by $f(x) = x^2 + 1$ and $g(x) = \frac{x}{x^2 + 1}$ then compute the derivative of fg.

(e) Find all possible values of constant k such that $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = k|x|$ is differentiable.

(f) If $f : [0, 1] \to \mathbb{R}$ is given by $f(x) = |2(2x - 1)^2 - 1|$ then compute $f'(\frac{1}{2})$

(g) Suppose $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = (x^2 + 1)^2 + 1$ then compute the minimum value of f.

(h) If $f, g : [0, 1] \to \mathbb{R}$ are given by $f(x) = x$ and $g(x) = 1 - x$ then compute $\int_0^1 (f(x) - g(x))dx$.

(i) If $h : \mathbb{R} \to \mathbb{R}$ is defined as $h(x) = x^3$. Compute $\int_{-100}^{100} h(x)dx$

(j) If a continuous function $f : [a, b] \to \mathbb{R}$ is differentiable on (a, b), with $f(a) = f(b)$ then which value is always contained in the range of $f' : (a, b) \to \mathbb{R}$

(k) If $f : \mathbb{R} \to \mathbb{R}$ is twice differentiable and $f'(a) = 0$ but a is neither a local minimum nor a local maximum of f then indicate the set of all possible values of $f''(a)$.

(l) Find the set of real roots of the polynomial $(x - 1)^{11} + x^5 + x^3 + 1$.

[12 × 3 = 36]

(2) Indicate *True* or *False* for each of the following statements and provide reasons supporting your answers. You will have to prove the statement if you indicate *True* and provide a counter example if you indicate *False*. Correct answer carries 1 mark each and a valid reasoning carries 3 marks each.

(a) Suppose $A = \{1, 2, \cdots, 8\}$, $B \subsetneq A$ and $f : A \to B$ is any arbitrary function then f is onto.

(b) Every function is a binary relation.

(c) A differentiable function $f : \mathbb{R} \to \mathbb{R}$ is continuous.

(d) If $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable then the derivative of their product is given by the product of their derivatives.

(e) If $f : \mathbb{R} \to \mathbb{R}$ is differentiable then at every local maxima of f, the derivative of f is zero.

(f) If $f : [a,b] \to \mathbb{R}$ is continuous. If A denotes the set of points at which f is differentiable and $A \subsetneq (a,b)$, then $\nexists\, c \in A$ such that $f(b) - f(a) = (b-a)f'(c)$

(g) Suppose $f, g : \mathbb{R} \to \mathbb{R}$ are functions such that $f(x) = (g(x) - 2)^2 + 1$, then the minimum value of f is 1

(h) If $f, g : [0,1] \to \mathbb{R}$ are continuous functions and $\int_0^1 f(x)dx \leq \int_0^1 g(x)dx$ then $f(x) \leq g(x)$ for every $x \in [0,1]$

(i) The average value of $f(x) = x^5$ over the interval $[0,6]$ is 1296.

[9 × (1+3) = 36]