Paper- 6.

Elements of Maths 1 - End-Sem Question Paper

MS LIS First Year

November 21, 2016

Instructions: Answer as much as you can. The maximum you can score is 60 marks. Marks corresponding to each question is indicated in bold. Maximum time allotted is 3 hrs.

(1) **[4]** Find all complex numbers z, such that $z\bar{z} = 25$ and $z + \bar{z} + (z - \bar{z})i = 14$ where $\bar{z}$ denotes the complex conjugate of z.

(2) **[6]** Suppose that there are 100 identical chocolates and 5 kids (numbered from 1 to 5). In how many ways can these chocolates be distributed among the kids such that kid numbered i receives at least i chocolates.

(3) **[6+6]** Suppose that there are n distinct pairs of shoes. Show that the number of ways (denote it by $f(n)$) in which the shoes can be paired such that no left shoe is paired with its correct right shoe

(a) satisfies the recurrence $f(n) = (n-1)[f(n-1) + f(n-2)]$ with $f(1) = 0$, $f(2) = 1$.

(b) and $f(n) = n!(\sum_{i=0}^{n} \frac{(-1)^i}{i!})$ for $n \geq 1$.

(4) **[6]** Use Binomial theorem or a combinatorial argument or otherwise prove:

$$\sum_{i=0}^{n}\sum_{j=0}^{n-i}\binom{n}{i}\binom{n}{j}\binom{n}{n-i-j} = \binom{3n}{n}$$

(5) **[4]** Compute $\sum_{i=0}^{n} i2^i$

(6) **[3]** Suppose that x, y and z are positive integers. Use AM-GM inequality or otherwise show that $(\frac{x}{y} + \frac{y}{z})(\frac{y}{z} + \frac{z}{x})(\frac{z}{x} + \frac{x}{y}) \geq 8$

(7) **[4]** Find the reflection of $(3, 5)$ w.r.t. the straight line $2x + 3y - 8 = 0$.

(8) **[6]** Suppose C_1 is a circle with radius 1 unit centred at $(1, 1)$. We recursively define C_i as the circle with smaller radius than that of C_{i-1} that touches C_{i-1} , X and Y axes for each $i \geq 2$. Let A_i denote the area of circle C_i. Compute $\sum_{i=1}^{\infty} A_i$

(9) **[3+3]** Prove that the circles $(x-1)^2 + (y-2)^2 = 9$ and $(x-3)^2 + (y-5)^2 = 1$ intersect. Find the equation of the straight line passing through the points of intersection of these circles.

(10) **[3+3]** Prove that the points $(1, 2)$, $(3, 4)$ and $(6, 8)$ are non-collinear. Find the equation of the circle passing through these points.

(11) **[4]** How many circles of radius 1 unit pass through $(1, 2)$ and $(3, 4)$? Justify.

(12) **[4]** Find the closest point on the circle $x^2 + y^2 = 4$ to the point $(0, 1)$. Justify your answer.