Indian Statistical Institute, Bangalore
M. Math.
Second Year, Second Semester
Mid-Term
Representation Theory and Linear Lie Groups Date : 09/03/07

Maximum marks: 100 Time: 3 hours

1. Let A be a unitary operator on a finite dimensional Hilbert space $\mathcal{H}$. Let α be a representation of the group of integers $\mathcal{Z}$ defined by $\alpha(n) = A^n$ for $n \in \mathcal{Z}$.

   (i) When is α irreducible? (ii) When is α cyclic?

   [15]

2. Show that an open subgroup of a topological group is also closed. Give an example of a topological group with a closed subgroup which is not open. [15]

3. Compute the Haar measure on R^*, the multiplicative group of non-zero real numbers.
   . [10]

4. Show that a compact group is unimodular. [15]

5. Give an example of a representation which is not unitarily equivalent to its contragradient representation. (Hint: Think of characters.) [5]

6. Let S_3 be the group of permutations of $\{1, 2, 3\}$. Let π be the representation of S_3 on $\mathbb{C}^3$, defined by

$$\pi(\sigma)(e_j) = e_{\sigma(j)} \quad \forall \sigma \in S_3,$$

   where $\{e_1, e_2, e_3\}$ is the standard ortho-normal basis of $\mathbb{C}^3$. (i) Compute the character of π. (ii) Find a non-trivial invariant subspace for π. (iii) Decompose π as a direct sum of irreducible representations. [20]

7. Show that every irreducible unitary representation of a compact group is finite dimensional. [20]

8. Show that every irreducible unitary representation of a compact group is equivalent to a subrepresentation of the right regular representation. [10]