

1. Let  $X$  denote the height of a randomly selected pipal tree. Assume, that  $X$  is normally distributed with unknown mean and standard deviation 16. Take a random sample of  $n = 16$  trees, so that, after setting the probability of committing a Type I error at  $\alpha$ , we can test the null hypothesis  $H_0 : \mu = 100$  against the alternative hypothesis that  $H_1 : \mu > 100$ .
  - (a) Graph the power function for  $\alpha = 0.05$ . What is the power of the hypothesis test if the true population mean were  $\mu = 108$ ?
  - (b) Graph the power function for  $\alpha = 0.01$ . What is the power of the hypothesis test if the true population mean were  $\mu = 108$ ?
2. Use the Neyman-Pearson Lemma to find the form of the critical region for the UMP level  $\alpha$  test for  $H_0$  vs  $H_1$  when
  - (a)  $X_1, X_2, \dots, X_n$  is i.i.d sample from Poisson distribution with mean  $\theta$  and  $H_0 : \theta = \theta_0$ ,  $H_1 : \theta = \theta_1, \theta > 0$ .
  - (b)  $X_1, X_2, \dots, X_n$  is i.i.d sample from Exponential distribution with mean  $\frac{1}{\theta}$  and  $H_0 : \theta = \theta_0$ ,  $H_1 : \theta = \theta_1, \theta_1 > \theta_0$ .