1. Show that the weak Poincare inequality is stable under rough isometry.

2. Consider, Γ the join of two copies of $\mathbb{Z}^3$ at the origin with natural weights. Let $f : V \to \mathbb{R}$ be such that it is 1 on one copy and -1 on the other copy. Using f show that the weak Poincare inequatlity does not hold on Γ.

3. Show that $\mathbb{Z}^d$ satisfied Weak Poincare Inequality.

4. Let (Γ, μ) be a weighted graph. Let $f \in L^2(V)$. Then for any $n \geq 0$,

$$0 \leq \| P^{n+1} f \|_2^2 - \| P^{n+2} f \|_2^2 \leq P^n f \|_2^2 - \| P^{n+1} f \|_2^2$$

5. Let (Γ, μ) be a weighted graph. Let $f \in L^2(V)$. Then for any $n \geq 0$,

$$\| f \|_2^2 - \| P^n f \|_2^2 \leq 2n\mathcal{E}(f, f)$$

6. Suppose Γ is $\mathbb{Z}^d$ equipped with weights μ_{xy} such that $\mu_{xy} \geq \frac{c_1}{d}$ for some $c_1 > 0$. Let the transition kernel of the random walk on Γ be denoted by $p_n(x, y)$. Then show that

$$p_n(x, y) \leq \frac{c_2}{n^{\frac{d}{2}}},$$

for some $c_2 > 0$.

7. Let Γ be two copies of $\mathbb{Z}^d$ joined at the origin. Let the transition kernel of the random walk on Γ be denoted by $p_n(x, y)$. Then show that

$$p_n(x, y) \leq \frac{c_2}{n^{\frac{d}{2}}},$$

for some $c_2 > 0$.