1. Let Γ_i for $i = 1, 2$ be graphs, with natural weights, which satisfy (N_{α_i}) respectively. Show that the join of Γ_1 and Γ_2 satisfies $(N_{\alpha_1 \wedge \alpha_2})$

2. Suppose Γ_1 and Γ_2 satisfy (N_α) then show that the join of Γ_1 and Γ_2 also satisfies (N_α).

3. Let (Γ, μ) be a weighted graph. Let $\lambda > 0$ and $\mu^\lambda = \lambda\mu$. If (Γ, μ) satisfies (N_α) with constant C_N then (Γ, μ^λ) satisfies (N_α) with $\lambda^{\frac{\alpha}{2}} C_N$

4. Let (Γ, μ) be a finite graph. Let $R_I(\Gamma)$ be the relative isoperimetric constant. Let $\mathcal{M}$ be a family of paths that cover[1] Γ. Let

$$\kappa(\mathcal{M}) = \max_{e \in E}\{\mu_e^{-1} \sum_{(x,y): e \in \gamma(x,y)} \mu_x \mu_y\}$$

   (a) Let $f : V \to \mathbb{R}$, show that

$$\mu(V) \min_\lambda \sum_{x \in V} \mid f(x) - \lambda \mid \mu_x \leq \sum_{y \in V} \sum_{x \in V} \mid f(x) - f(y) \mid \mu_x \mu_y$$

   (b) Let $f : V \to \mathbb{R}$, show that

$$\sum_{y \in V} \sum_{x \in V} \mid f(x) - f(y) \mid \mu_x \mu_y \leq \kappa(\mathcal{M}) \parallel \nabla f \parallel_1$$

   (c) Show that

$$R_I(\Gamma) \geq \frac{\mu(V)}{\kappa(\mathcal{M})}$$

5. Consider, $\Gamma = \mathbb{Z}^d$.

   (a) Let $Q = \{1, 2 \ldots, R\}^d$ be the cube in $\mathbb{Z}^d$. Let R_I denote the relative isoperimetric constant for Q. Show that Show that $R_I(Q) \geq \frac{c_d}{R}$.

   (b) Show that $\mathbb{Z}^d$ satisfies I_d.

[1] $\mathcal{M}$ is said to cover Γ if for each distinct pair $x, y \in V$ there is a path $\gamma \equiv \gamma(x, y) \in \mathcal{M}$ from x to y.