1. Show that $\mathbb{Z}^3$ is transient by using the flow[1] on $\mathbb{Z}^3_+$ given by;

$$I_{(x,y,z),(x+1,y,z)} = \frac{x+1}{C}, I_{(x,y,z),(x,y+1,z)} = \frac{y+1}{C}, I_{(x,y,z),(x,y,z+1)} = \frac{z+1}{C},$$

with $C = (x+y+z+1)(x+y+z+2)(x+y+z+3)$. Further conclude that $\mathbb{Z}^d$ is transient for $d \geq 3$.

2. Let $\mathbb{T}$ be a rooted binary tree, with root ρ, as discussed in class. Let $\mathbb{T}_n$ be the set of 2^n points at a distance n from the root ρ.

   (a) Using the flow I given by

   $$I_{x_n,x_{n+1}} = 2^{-(n+1)} \text{ for } x_n \in \mathbb{T}_n, x_{n+1} \in \mathbb{T}_{n+1} \text{ and } x_n \sim x_{n+1}$$

   show that $\mathrm{R}_{\text{eff}}(\rho, \mathbb{T}_n) \leq 1 - 2^{-n}$

   (b) Using the function f given by

   $$f(x) = \frac{1 - 2^{-d(\rho,x)}}{1 - 2^{-n}}$$

   show that $\mathrm{R}_{\text{eff}}(\rho, \mathbb{T}_n) \geq 1 - 2^{-n}$

   (c) Show that $\mathbb{T}$ is transient.

   (d) Suppose we allow weights on $\mathbb{T}$ to be given by

   $$\mu_{x_n,x_{n+1}} = \frac{1}{r_n} \text{ for } x_m \in \mathbb{T}_n, x_{n+1} \in \mathbb{T}_{n+1} \text{ and } x_n \sim x_{n+1},$$

   where r_n is positive sequence of numbers. Decide if $\mathbb{T}, \mu$ is transient or not.

3. Let $V_n \uparrow V$. Then if $B_0, B_1 \subset V$ show that

$$\mathrm{C}_{\text{eff}}(B_0, B_1) = \lim_{n\to\infty} \mathrm{C}_{\text{eff}}(B_0 \cap V_n, B_1 \cap V_n; V_n)$$

4. Let $I \in \bar{\mathcal{I}}_o(B_0, B_1)$ and let $I^n \in \mathcal{I}_o(B_0, B_1)$, with $E[I - I^n, I - I^n] \to 0$. Then $I^n_{xy} \to I_{xy}$ for each $x, y \in V$. Show that $\mathrm{Div}I(x) = 0$ for $x \in V \setminus B_0 \cup B_1$. In addition if $B_0 \cup B_1$ is finite then I satisfies $\sum_{x\in B_0} \mathrm{Div}I(x) = 1$ and $\sum_{x\in B_0} \mathrm{Div}I(x) = -1$

5. Show that $1 \in H^2_0 \iff H^2_0 = H^2$

[1] Verify $\mathrm{Div}I(x) = 0$ for all $x \neq 0$ and $E(I, I) < \infty$