Due: Thursday, February 20th, 2014
Problem to be turned in: 2, 3(c)

1. Suppose that X and Y are random variables with joint probability density

$$f(x,y) = \begin{cases} c(xy+2) & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

where $c > 0$ is a constant to ensure that f is a probability density function. Find the conditional distribution $Y \mid X = x$ for $0 \le x \le 1$.

2. Let $R > 0$. Suppose that (X, Y) is a point chosen uniformly in ball of radius R around the origin in $\mathbb{R}^2$. Find the conditional distribution $Y \mid X = x$ for $0 \le x \le R$.

3. Let X_1 and X_2 be independent Normal random variables with mean 0 and variance 1.
   (a) Suppose $Y_1 = X_1$ and $Y_2 = \rho X_1 + \sqrt{1-\rho^2} X_2$. Find the joint p.d.f. of (Y_1, Y_2).
   (b) Suppose $Y_1 = a + bX_1$ and $Y_2 = c + dX_2$, with $c, d > 0$. Find the joint p.d.f. of (Y_1, Y_2).
   (c) Suppose $0 \le R < \infty, -\pi < \Theta < \pi$ are random variables such that $X_1 = R\cos(\Theta)$ and $X_2 = R\sin(\Theta)$. Find the joint p.d.f. of (R, Θ).

4. Suppose X_1 and X_2 are independent Exponential (λ) random variables. Find the conditional distribution of X_1 given $X_1 + X_2 = z$ for some $z > 0$.

5. Let X_1 and X_2 be random variables with joint probability density function f. Suppose $f(x, y) > 0$ if and only if $x > 0$ and $y > 0$. Let $Y_1 = \frac{X_2}{X_1}$ and $Y_2 = X_1 + X_2$.
   (a) Find the joint p.d.f of Y_1, Y_2.
   (b) Further if X_1 and X_2 are independent $\Gamma(\alpha, \lambda)$ random variables show that Y_1 and Y_2 are independent. Are there any other examples of f where this happens ?

6. Suppose X_1 and X_2 are independent $\Gamma(\alpha, \lambda)$ random variables. Find the distribution of $\frac{X_1}{X_1+X_2}$.