Due: Thursday, February 6th, 2014
Problem to be turned in: 1

1. Suppose X is a uniform random variable in the interval $(0, 1)$ and Y is an independent exponential(2) random variable. Find the distribution of $Z = X + Y$.

2. Let X and Y have a joint probability density function given by

$$f(x,y) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq y \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute the marginal probability density functions for X and Y.

   (b) Compute $P\big(X \leq 1,\ Y \leq \frac{1}{2}\big)$.

3. Sunita makes cuts at two points selected at random on a piece of lumber of length L. Find the distribution of M, the length of the middle piece. What is the expected value of the length of the middle piece?

4. Suppose X, Y are independent random variables each being distributed as Normal with mean 0 and variance 1. Find the $P(X^2 + Y^2 \leq 4)$?

5. Let

$$f(x,y) = \begin{cases} \eta(x-y)^{\gamma} & \text{if } 0 \leq x < y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) For what values of γ can η be chosen so that f be a joint probability density function of X, Y.

   (b) In cases as in (a), what are the values of η ?

   (c) In such cases as in (a) and (b)

      i. Find the marginal densities of X, and Y.

      ii. Find the distribution of $X + Y$.

6. Let $n \geq 1$ and $X_1, X_2, X_3, \ldots, X_n$ be independent and identically distributed $N(0, 1)$ random variables.

   (a) Find the distribution of $\frac{1}{n}\sum_{i=1}^{n} X_i$

   (b) Find the distribution of $Y_i = X_i^2$ for $i = 1, \ldots n$.

   (c) Find the distribution of $\sum_{i=1}^{n} Y_i$.