**Due: Thursday, January 30th, 2014**
*Problem to be turned in: 1*

1. Suppose that $X$ and $Y$ are random variables with joint probability density

$$f(x,y) = \begin{cases} \frac{4}{5}(xy+1) & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0 & \textit{otherwise} \end{cases}$$

   (a) Compute the marginal densities $f_1(x)$ and $f_2(y)$.
   (b) Are $X$ and $Y$ independent ?
   (c) Calculate the means $\mu_X$, $\mu_Y$, the variances $\sigma_X^2, \sigma_Y^2$ and the covariance $\sigma_{XY}$.
   (d) Calculate $E(X^2 + Y^2)$.

2. Suppose $Y_1$ and $Y_2$ have a joint probability density function given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq y_2 \leq y_1 \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute the marginal probability density functions for $Y_1$ and $Y_2$.
   (b) Compute $P\big(Y_1 \leq 1,\ Y_2 \leq \frac{1}{2}\big)$.

3. Let $D = \{(x,y) : x^2 \leq y \leq x\}$. A point $(X,Y)$ is chosen uniformly from $D$. Find the joint probability density function of $X$ and $Y$.

4. Let $k$ be a positive number. Consider the joint p.d.f of $X_1$ and $X_2$ to be given by

$$f(x_1, x_2) = \begin{cases} k & \text{if } 0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1 \text{ and } 2x_2 \leq x_1 \\ 0 & \text{otherwise} \end{cases}$$

   Let $U = X_1 - X_2$.

   (a) Find the probability density function for $U$.
   (b) Find $E(U)$.

5. Suppandi and Meera plan to meet at Gopalan Arcade between 7pm and 8pm. They decide to reach at a time (independent of each other) uniformly between 7pm and 8pm and wait for 15 minutes for the other person. Find the probability that they will meet ?