Probability 1 **Quiz 5** **Semester I 2016/17**

Your name : Solution

1. Suppose that the number of earthquakes that occur in a year in California has a Poisson distribution with parameter 2. Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is $\frac{1}{3}$. Find the probability that there will be exactly 20 earthquakes of magnitude at least 5 in a year.

Solution to Quiz 5

Let number of earthquakes is Poisson(λ) distributed.

Let p denote the probability that a given earthquake reading is higher than the specified limit in the problem.

For the event exactly m earthquake with specified readings to occur, note that at least m earthquake must occur.

Let E_n denote the event that n earthquake occur.

Let Y denote the number of earthquakes with specied reading. Note that Y is a random variable.

Now $$\mathbb{P}[Y=m] = \mathbb{P}\left[\{Y=m\} \cap \bigcup_{n=0}^{\infty} E_n\right] = \mathbb{P}\left[\bigcup_{n=0}^{\infty} \{Y=m\} \cap E_n\right]$$

$$= \sum_{n=0}^{\infty} \mathbb{P}\left[\{Y=m\} \cap E_n\right] \quad \left\{ E_i \cap E_j = \phi \;\; \forall\, i \neq j \right.$$

Clearly $\mathbb{P}[\{Y=m\} \cap E_n] = 0 \;\; \forall\, n < m$.

Also, given the event E_n, for $n \geq m$

$$\mathbb{P}\left[\{Y=m\} \mid E_n\right] = \binom{n}{m} p^m (1-p)^{n-m}$$

So, $$\mathbb{P}[\{Y=m\} \cap E_n] = \mathbb{P}[Y=m \mid E_n]\, \mathbb{P}[E_n]$$

By assumption of problem $\mathbb{P}[E_n] = \dfrac{e^{-\lambda} \lambda^n}{n!}$

So, $$\mathbb{P}[Y=m] = \sum_{n=m}^{\infty} \binom{n}{m} p^m (1-p)^{n-m} \cdot \frac{e^{-\lambda} \lambda^n}{n!}$$

$$= \frac{p^m e^{-\lambda}}{m!\,(1-p)^m} \sum_{n=m}^{\infty} \frac{1}{(n-m)!} (\lambda(1-p))^n = \frac{e^{-\lambda p} (\lambda p)^m}{m!} \quad \left\{ \begin{array}{l} \text{By using} \\ e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} \end{array} \right.$$

Thus putting $\lambda = 2,\, p = \frac{1}{3},\, m = 20$ or $\lambda = 1,\, p = 1/4,\, m = 10$ we get the answers in two different problems.