**Probability 1** **Quiz 4** **Semester I 2016/17**

**Ground Rules:** Time allowed is 15 minutes, individual work only and closed book test.

Your name Solution

**Score :**

1. Suppose we toss two fair dice. Let $E_1$ denote the event that the sum of the dice is six. $E_2$ denote the event that sum of the dice equals nine Let $F$ denote the event that the first die equals three. Is $E_1$ independent of $F$? Is $E_2$ independent of $F$?

The event $E_1, E_2$ is represented by set

$$E_1 = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}, \quad \mathbb{P}[E_1] = \frac{|E_1|}{|S|} = \frac{5}{36}$$

$$E_2 = \{(3,6),(4,5),(5,4),(6,3)\} \quad \mathbb{P}[E_2] = \frac{|E_2|}{|S|} = \frac{4}{36}$$

$$F = \{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\} \quad \mathbb{P}[F] = \frac{|F|}{|S|} = \frac{6}{36}$$

Now $\mathbb{P}[E_1 \cap F] = \frac{|E_1 \cap F|}{|S|}$ $\left\{ \text{Where sample space } S = \{1,..,6\} \times \{1,..,6\},\ |S| = 36 \right\}$

$$= 1/36$$

$$\mathbb{P}[E_2 \cap F] = \frac{|E_2 \cap F|}{|S|} = \frac{1}{36}$$

Since $\mathbb{P}[E_1 \cap F] \neq \mathbb{P}[E_1]\,\mathbb{P}[F]$

and $\mathbb{P}[E_2 \cap F] \neq \mathbb{P}[E_2]\,\mathbb{P}[F]$

neither of $E_1$ and $E_2$ is independent of $F$.

2. Suppose that airplane engines operate independently in flight and fail with probability $\frac{1}{2}$. A plane makes a safe flight if at least half of its engines are running. Airline A has a four–engine plane and Airline B has a two–engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight?

Airline A, A has 4 engines

A makes a successful flight if number of working engine is greater equal 2, so either 2, 3, or 4

$$\mathbb{P}[\text{exactly } k \text{ engine work}] = \binom{4}{k}\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{4-k} = \binom{4}{k}\frac{1}{16}$$

So, Probability that A makes a successful flight

$$= \frac{1}{16}\left(\binom{4}{2}+\binom{4}{3}+\binom{4}{4}\right) = \frac{11}{16}$$

For Flight B: B has two engines.

B has a successful flight if at least 1 engine work,

$P[\text{exactly one engine work}] = \binom{2}{1}\frac{1}{2}\frac{1}{2}$

$P[\text{exactly two engine work}] = \binom{2}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^0$

So, Probability that B ~~has~~ makes successful flight

$$= \frac{1}{4}\left(\binom{2}{1}+\binom{2}{2}\right) = \frac{3}{4} = \frac{12}{16}$$

Since $\frac{12}{16} > \frac{11}{16}$

Flight B has higher probability of making a successful flight.

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**Score :**

1. Suppose we toss two fair dice. Let $E_1$ denote the event that the sum of the dice is six. $E_2$ denote the event that sum of the dice equals ~~six~~ 7. Let $F$ denote the event that the first die equals four. Is $E_1$ independent of $F$? Is $E_2$ independent of $F$?

The event $E_1, E_2$ is represented by set

$E_1 = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ $P[E_1] = \frac{|E_1|}{|S|} = \frac{5}{36}$

$E_2 = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$ $P[E_2] = \frac{|E_2|}{|S|} = \frac{6}{36}$

$F = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$ $P[F] = \frac{|F|}{|S|} = \frac{6}{36}$

$P[E_1 \cap F] = \frac{|E_1 \cap F|}{|S|} = \frac{1}{36}$ { where $S$ is the sample space $S = \{1..6\} \times \{1..6\}$, $|S| = 36$ }.

$P[E_2 \cap F] = \frac{|E_2 \cap F|}{|S|} = \frac{1}{36}$

$P[E_1]\,P[F] = \frac{5}{36} \times \frac{6}{36} \neq \frac{1}{36} = P[E_1 \cap F]$

So, $E_1$ is not independent of $F$.

$P[E_2]\,P[F] = \frac{6}{36} \times \frac{6}{36} = \frac{1}{36} = P[E_2 \cap F]$.

So, $E_2$ is independent of $F$.

2. Suppose that airplane engines operate independently in flight and fail with probability $\frac{3}{4}$. A plane makes a safe flight if at least half of its engines are running. Air line A has a four–engine plane and Airline B has a two–engine plane for a flight from Bangalore to Delhi. Which airline has the higher probability for a successful flight?

Airline A: A has 4 engines:

A makes successful flight number of working engine is greater equal 2, so either 2, 3, 4.

$P[\text{exactly } k \text{ engine work}] = \binom{4}{k}\left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{4-k}$

So, probability that A makes a successful flight

$$= \binom{4}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 + \binom{4}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) + \binom{4}{4}\left(\frac{1}{4}\right)^4$$

$$= \frac{67}{256}$$

<u>For Flight B:</u> B has two engines

B makes a successful flight if at least 1 engine work, So either 1 engine or both engine work:

$$P[\text{exactly } k \text{ engine work}] = \binom{2}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{2-k}$$

So, probability that B makes a successful flight

$$= \binom{2}{1}\frac{1}{4}\times\frac{3}{4} + \binom{2}{2}\left(\frac{1}{4}\right)^2$$

$$= \frac{7}{16} = \frac{112}{256}$$

Since $\frac{112}{256} > \frac{67}{256}$

Flight B has higher probability of making a successful flight.

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