Probability 1 **Quiz 3** **Semester I 2016/17**

Ground Rules: Time allowed is 15 minutes, individual work only and closed book test.

Your name Solution **Score :**

1. A box contains 10 black balls and 15 white balls. A sample of three balls is drawn at random with each drawn ball being discarded (not put back into the box) after it is drawn. Let A_3 denote the event that the ball drawn on the 3rd draw is white. Find $P(A_3)$.

$$A_3 = \{A_3 \cap A_1 \cap A_2\} \cup \{A_3 \cap A_1 \cap A_2^c\} \cup \{A_3 \cap A_1^c \cap A_2\} \cup \{A_3 \cap A_1^c \cap A_2^c\}$$

where $A_i = \{$ ball drawn on i^{th} draw is white $\}$ for $i = 1, 2$

$$\therefore P(A_3) = P(A_3 \cap A_1 \cap A_2) + P(A_3 \cap A_1 \cap A_2^c) + P(A_3 \cap A_1^c \cap A_2) + P(A_3 \cap A_1^c \cap A_2^c)$$

$$= P(A_3 | A_1 \cap A_2) P(A_2 | A_1) P(A_1) + P(A_3 | A_1 \cap A_2^c) P(A_2^c | A_1) P(A_1) + P(A_3 | A_1^c \cap A_2) P(A_2 | A_1^c) P(A_1^c) + P(A_3 | A_1^c \cap A_2^c) P(A_2^c | A_1^c) P(A_1^c)$$

As we are doing sampling without replacement

$$P(A_3) = \frac{13}{23} \cdot \frac{14}{24} \cdot \frac{15}{25} + \frac{14}{23} \cdot \frac{10}{24} \cdot \frac{15}{25} + \frac{14}{23} \cdot \frac{15}{24} \cdot \frac{10}{25} + \frac{15}{23} \cdot \frac{9}{24} \cdot \frac{10}{25}$$

$$= \frac{1}{23 \cdot 24 \cdot 25} [15 \cdot 14 \cdot 13 + 15 \cdot 14 \cdot 10 + 15 \cdot 14 \cdot 10 + 15 \cdot 9 \cdot 10]$$

$$= \frac{1}{23 \cdot 24 \cdot 25} [15 \cdot 14 \cdot 23 + 15 \cdot 10 \cdot 23]$$

$$= \frac{1}{24 \cdot 25} [15 \cdot 14 + 15 \cdot 10] = \frac{15 \cdot 24}{24 \cdot 25}$$

$$= \frac{15}{25} = \frac{3}{5} \quad \square$$