Due: Thursday, March 31st 2016
Problem to be turned in: 7

1. Let $X_n \xrightarrow{d} X$. Show that $X_n^2 \xrightarrow{d} X^2$.
2. Let $X_1, X_2, \ldots, X_n, \ldots$ be i.i.d. random variables with finite second moment. Define
$$Y_n = \frac{2}{n(n+1)} \sum_{i=1}^{n} iX_i.$$
Show that $Y_n \xrightarrow{p} E(X_1)$ as $n \to \infty$.
3. Let $X_n \xrightarrow{d} X$ and let F denote the distribution function of X. Let a be a continuity point of F. Show that $P(X_n = a) \to 0$.
4. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables with $X_1 \sim$, Exponential (1). Find
$$\lim_{n\to\infty} P\left(\frac{n}{2} - \frac{\sqrt{n}}{2\sqrt{3}} \leq \sum_{i=1}^{n}[1 - \exp(-X_i)] \leq \frac{n}{2} + \frac{\sqrt{n}}{2\sqrt{3}}\right).$$
5. Let $a_n = \sum_{k=0}^{n} \frac{n^k}{k!} e^{-n}$, $n \geq 1$. Using the Central Limit Theorem evaluate $\lim_{n\to\infty} a_n$.
6. Suppose that the weight of open packets of daal in a home is uniformly distributed from 200 to 600 gms. In random survey of 64 homes, find the (approximate) probability that the total weight of open boxes is less than 25 kgs.
7. Toss a fair coin 400 times. Use the central limit theorem to
   (a) find an approximation for the probability of at most 190 heads.
   (b) find an approximation for the probability of at least 70 heads.
   (c) find an approximation for the probability of at least 120 heads.
   (d) find an approximation for the probability that the number of heads is between 140 and least 160.