

Due: Thursday, March 24th 2016

Problem to be turned in:

1. Suppose X_1, X_2, X_3 be random variables such that $X_i \sim \text{Normal}(i, i^2)$. Find f such that
 - (a) $f(X_1, X_2, X_3) \sim \chi_3^2$
 - (b) $f(X_1, X_2, X_3) \sim t_2$
 - (c) $f(X_1, X_2, X_3) \sim F(1, 2)$
2. Let $m \leq n$. Suppose $X \sim \chi_m^2$ and $Y \sim \chi_n^2$. Show that $P(X \geq a) \leq P(Y \geq a)$ for all $a \in \mathbb{R}$.
3. Let X_1, X_2, \dots, X_n be i.i.d. X such that $\text{Var}[X] = \sigma^2$. Let Y_1, Y_2, \dots, Y_n be i.i.d. Y such that $\text{Var}[Y] = \sigma^2$. Find a suitable n such that

$$P(|\bar{X}_n - \bar{Y}_n| < \frac{\sigma}{5}) \approx 0.99.$$

4. Let X_1, X_2, \dots, X_{100} be i.i.d. X such that $\text{Var}[X] = 16$. Find $\beta < \alpha > 0$ such that

$$P(\beta < \bar{X}_{100} - \mu < \alpha) \geq 0.90.$$

using: (a) Central limit Theorem and (b) Chebychev's Inequality.