

Due: Thursday, January 28th 2016

Problem to be turned in: 1,2

1. Let $a > 0$ and X be a random variable with the p.d.f given by

$$f(x) = \begin{cases} \frac{a}{x^{a+1}} & 1 \leq x < \infty \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the distribution of $X_1 = X^2$
 (b) Find the distribution of $X_2 = \frac{1}{X}$
 (c) Find the distribution of $X_3 = \ln(X)$

In the above exercises please assume that the transformation function is defined as above when the p.d.f of X is positive and zero otherwise. The random variable X is said to have Pareto (α) distribution.

2. Let X, Y be two independent standard Normal random variables.

- (a) Find the distribution of $R = \sqrt{X^2 + Y^2}$
 (b) Find the distribution of $T = aX + bY$
 (c) Find the distribution of $C = \frac{|X|}{|Y|}$

3. Let X, Y be two independent continuous random variables. Can you find an expression for the density of $Z = XY$?

4. Let X and Y be two random variables with the joint p.d.f given by

$$f(x, y) = \begin{cases} \lambda^2 e^{-\lambda y} & 0 \leq x \leq y \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal distribution of X and Y . Find the conditional distribution of $Y \mid |X = x$ for some $x > 0$

5. Let $a, b > 0$. Let $X \sim \text{Gamma}(a, b)$ and $Y \sim \text{Exponential}(X)$. Find the joint density of X and Y ; the marginal density of Y ; and the conditional density of $X \mid Y = y$.
6. Consider three random variables X_1, X_2, X_3 , independent and identically distributed Uniform $(0, 1)$ random variables. Let $A = X_1 X_3$ and $B = X_2^2$. Find the $P(A < B)$.