Due: Thursday, April 14th 2016
Problem to be turned in:

1. Let $X : S \to \mathbb{R}$ be a symmetric random variable. Then show that the characteristic function $\phi_X(t) \in \mathbb{R}$ for all $t \in \mathbb{R}$.

2. Let $\alpha \in \mathbb{C}, \beta \in \mathbb{C}$ and $X : S \to \mathbb{R}$ be a random variable. Let $Y = \alpha X + \beta$. Then show that the characteristic function of Y is given by $\phi_Y(t) = e^{it\beta}\phi_X(t\alpha)$ for all $t \in \mathbb{R}$, with $\phi_X(\cdot)$ being the characteristic function of X.

3. Verify the formulae for the characteristic functions of the random variable X given below:-

Distribution of X	**Characteristic Function** $\phi_X(t), t \in \mathbb{R}$
Bernoulli (p)	$1 - p + pe^{it}$
Binomial (n, p)	$(1 - p + pe^{it})^n$
Uniform $(\{1, 2, \ldots, n\})$	$\frac{e^{it}(1-e^{it})}{n(1-e^{int})}$
Poisson (λ)	$e^{\lambda}(e^{it-1})$
Uniform (a, b)	$\frac{e^{ibt}-e^{iat}}{i(b-a)t}$
Normal (m, σ^2)	$e^{-imt-\frac{\sigma^2 t^2}{2}}$