**Due: Tuesday, February 11th, 2010**

1. A line from 4 to 10 has midpoint 7. A point $x$ is chosen at random on this line. Find the probability that the line segments $[4, x]$, $[x, 10]$ and $[4, 7]$ can be joined to form a triangle.

2. Of the people who enter a blood bank to donate blood, 1 in 3 have type $O^+$ blood and 1 in 15 have type $O^-$ blood. For the next three people entering the blood bank, let $X$ denote the number with type $O^+$ blood and $Y$ the number with type $O^-$ blood. Find the probability distributions for $X$, $Y$ and $X + Y$.

3. The length of time $X$ required to complete a certain task is an exponentially distributed random variable with mean 10 hours. The cost $C$ of completing this task is

$$C = 100 + 40X + 3X^2$$

   (a) Find the mean and variance of $C$.

   (b) Find the probability that $C$ exceeds \$2000.

4. Suppose that $X$ has an exponential distribution with mean $\theta$. We showed in class that, for all $s, t > 0$,

$$P(X > s + t|X > s) = P(X > t).$$

   This is called the "memoryless property" of the exponential distribution. Suppose that $Y$ is another positive continuous memoryless random variable. Show that $Y$ has exponential distribution with some mean $\theta > 0$.

5. The joint probability density function of $X$ and $Y$ is given by

$$f(x, y) = c(y^2 - x^2)e^{-y} \qquad -y \le x \le y,\ 0 < y < \infty$$

   (a) Find $c$.

   (b) Find the marginal densities of $X$ and $Y$.

   (c) Find $E(X)$.

   (d) Find $P\big(X < \frac{1}{2}Y\big)$.

6. Thieves stole four animals at random from a farm that had seven sheep, eight goats and five burros. Calculate the joint probability function of the number of sheep and goats stolen.

7. A farmer makes cuts at two points selected at random on a piece of lumber of length $\ell$. What is the expected value of the length of the middle piece?

8. Let $R$ be the region between $y = x$ and $y = x^2$. A random point $(X, Y)$ is selected from $R$. Find the joint probability density function of $X$ and $Y$.

9. The joint distribution of amount of pollutant emitted from a smokestack without a cleaning device $(X_1)$ and with a cleaning device $(X_2)$ is given by

$$f(x_1, x_2) = \begin{cases} k & \text{if } 0 \le x_1 \le 2, 0 \le x_2 \le 1 \text{ and } 2x_2 \le x_1 \\ 0 & \text{otherwise} \end{cases}$$

   The reduction in amount of pollutant emitted due to the cleaning device is given by $U = X_1 - X_2$.

   (a) Find the probability density function for $U$.

(b) Find $E(U)$.

10. $Y_1$ and $Y_2$ have a joint probability density function given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq y_2 \leq y_1 \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the marginal probability density functions for $Y_1$ and $Y_2$.

(b) Compute $P\big(Y_1 \leq 1,\ Y_2 \leq \frac{1}{2}\big)$.