

Due: Thursday, August 6th, 2009

Problem to be turned in: 1,3.

1. Consider a deck of 50 cards. Each card has one of 5 colors (black, blue, green, red, and yellow), and is printed with a number (1,2,3,4,5,6,7,8,9, or 10) so that each of the 50 color/number combinations is represented exactly once. A **hand** is produced by dealing out five different cards from the deck. The order in which the cards were dealt does not matter.
 - (a) How many different hands are there?
 - (b) How many hands consist of cards of identical color?
 - (c) How many hands contain exactly three cards with one number, and two cards with a different number?
 - (d) How many hands contain two cards with one number, two cards with a different number, and one card of a third number?
2. A box contains 500 envelopes, of which 50 contain Rs 100 in cash, 100 contain Rs 50 in cash and 350 contain Rs 10. An envelope can be purchased at Rs 25 from the owner, who will pick an envelope at random and give it to you. Write down the sample space for the net money gained by you. What is the probability that the first envelope purchased contains less than 100 Rs ?
3. Sheela has lost her key to her room. The security officer gives her 50 keys and tells her that one of them will open her room. She decides to try each key successively and notes down the number of the attempt at which the room opens. Describe the sample space for this experiment and also decide on the probability of each outcome.
4. (*Feller Volume 1 page 56, problem 20*) From a population of n elements a sample of size r is taken. Find the probability that none of the N prescribed elements will be included in the sample, assuming the sampling to be (a) without, (b) with replacement.
5. Suppose that r indistinguishable balls are placed in n distinguishable boxes so that each distinguishable arrangement is equally likely. Find the probability that no box will be empty.
6. If n balls are placed at random into n boxes, find the probability that a) none of the boxes are empty and b) that exactly one box remains empty ?
7. Suppose that n sticks are broken into two one long and one short piece. The $2n$ pieces are now arranged into n pairs from which new sticks are formed. Find the probability (a) that the pieces will be joined in the original order, (b) that all long pieces are paired with short pieces.
8. Suppose a population of size N contains G Green people and B Blue people, with $N = G + B$. A sample of size n , $1 \leq n \leq N$ is taken. Find the probability that g , $1 \leq g \leq \min(n, G)$, Green people are selected in the sample, assuming the sampling to be (a) without, (b) with replacement.

Ahead in class:- *Polya Urn scheme*– An urn contains b black balls and r red balls. A ball is drawn at random. The ball is replaced into the urn along with c balls of its colour and d balls of the opposite colour. Then another random ball is drawn and the procedure is repeated. Here c and d are integers. A simple question one can try is :- At the second draw what is the probability of obtaining a black ball ?

The above model was used by G. Polya to model aftereffects from an accident. (See Feller Volume 1 page 119). A