Due: January 19th, 2008
Problems to be turned in: 10,13,17

1. A die is rolled and then a coin is tossed. Describe the sample space for this experiment.
2. Suppose A and B are subsets of a sample space Ω.
   (a) Show that $(A - B) \cup B = A$ when $B \subset A$.
   (b) Show by example that the equality doesn't always hold if B is not a subset of A.
3. Consider a deck of 50 cards. Each card has one of 5 colors (black, blue, green, red, and yellow), and is printed with a number (1,2,3,4,5,6,7,8,9, or 10) so that each of the 50 color/number combinations is represented exactly once. A **hand** is produced by dealing out five different cards from the deck. The order in which the cards were dealt does not matter.
   (a) How many different hands are there?
   (b) How many hands consist of cards of identical color?
   (c) How many hands contain exactly three cards with one number, and two cards with a different number?
   (d) How many hands contain two cards with one number, two cards with a different number, and one card of a third number?
4. Consider the sample space $\Omega = \{a, b, c, d, e\}$. Given that $\{a, b, e\}$, and $\{b, c\}$ are both events, what other events are implied by taking unions, intersections, and compliments?
5. The résumés of two male applicants for a college teaching position in psychology are placed in the same file as the résumés of two female applicants. Two positions become available and the first, at the rank of assistant professor, is filled by selecting one of the four applicants at random. The second position, at the rank of instructor, is then filled by selecting at random one of the three remaining applicants.
   (a) List the elements of the sample space S.
   (b) List the elements of the event A that the position of assistant professor is filled by a male applicant.
   (c) List the elements of the event B that exactly one of the two positions is filled by a male applicant.
   (d) List the elements of the event C that neither position was filled by a male applicant.
   (e) Sketch a Venn diagram to show the relationship among the events A, B, C and S
6. Assuming the axioms of Probability show the following:
   (a) If A is an event and $\bar{A}$ its complement, then $P(\bar{A}) = 1 - P(A)$
   (b) If A, B and C are events then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
   (c) Suppose that E and F are events.If $E \subset F$, then $P(F - E) = P(F) - P(E)$. Show that $P(E \cap F) \leq P(E \cup F) \leq P(E) + P(F)$.
7. Suppose we toss two fair dice: Let E_1 denote the event that the sum of the dice is six. E_2 denote the event that sum of the dice equals seven. Let F denote the event that the first die equals four. Is E_1 independent of F ? Is E_2 independent of F ?
8. Suppose that each of three women at a party throws here hat into the center of the room. The hats are first mixed up and then each one randomly selects a hat. What is the probability that none of the three women selects her own hat ?
9. Assuming that birthdays are spread evenly among the days in a standard 365 day year (that is, a person is as likely to have any one day as any other as his/her birthday), compute the probability that amongst a group of 40 people there are atleast 2 people having the same birthday ?

10. It is estimated that 0.8% of a large shipment of eggs to a certain supermarket are cracked. The eggs are packaged in cartons, each with a dozen eggs, with the cracked eggs being randomly distributed. A customer buys 10 cartons. Then she notes down the number of cartons with cracked eggs. Write down the probability space for this experiment.

11. Suppose that an airplane engine will fail, when in flight, with probability $1-p$ independently from engine to engine; suppose that the airplane will make a successful flight if atleast 50 percent of its engines remain working. For what values of p is a four-engine plane preferable to a two-engine plane ?

12. A fair die is rolled repeatedly.

    (a) What is the chance that the first 6 appears before the tenth roll.

    (b) What is the chance that the third 6 appears on the tenth roll.

    (c) Given that there were six 6's among the first twenty rolls, what is the chance of seeing three 6's among the first ten rolls.

13. A box contains M balls, of which W are white. A sample of n balls, with $n \leq W$ and $n \leq M-W$, is drawn at random and without replacement. Let A_j, where $j = 1, 2, \cdots, n$, denote the event that the ball drawn on the j^{th} draw is white. Find $P(A_1)$, $P(A_2)$ and $P(A_3)$. Guess what $P(A_j)$ is.

14. A population of 100 voters contain 60 in favour of the "Ulta Pulta party" and the rest who are opposed.An opinion poll selects a random sample of 4 voters from this population as follows.One person is picked at random from 100 voters, then another at random from the remaining 99, and so on, till 4 people have been picked.

    (a) What is the probability that there will be no one in favour of "Ulta Pulta party" in the sample ?

    (b) What is the probability that there will be at least one person in favour "Ulta Pulta party" in the sample?

    (c) How large a sample must be taken for there to be a 99% chance that the majority of voters in the sample will favour "Ulta pulta" party ?

15. A box contains three coins. One of the coins is fair and lands heads with probability $\frac{1}{2}$ when tossed. The other two coins land on heads with probabilities $\frac{1}{2}$ and $\frac{2}{3}$ respectively when tossed.

    (a) One of three coins is chosen at random and tossed. What is the chance that heads will be the outcome ?

    (b) Given that the outcome is a tail, what is the chance that the fair coin was chosen ?

16. Consider two machines A and B each producing the same items. Each machine produces a large number of these items every day. However, production per day from machine B, being newer, is twice that of A. Further the rate of defectives is 1% for B and 2% for A. The daily output of the machines is combined and then a random sample of size 12 taken. Find the probability that the sample contains 2 defective items. What assumptions are you making ?

17. Construct an example of three events such that they are pairwise independent but not independent.

18. There are n letters addressed to n people at n different addresses. The n addresses are typed on n envelopes. A disgruntled union worker shuffles the letters and puts them in envelopes in random order, one letter per envelope.

    (a) Find the probability that at least one letter is put in a correctly addressed envelope.

    (b) What is this probability approximately, for large n ?