Problems due: 1,2
Due Date: Friday October 24th, 2014.

1. Solve (graphically) the linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & x_1 + x_2 \\ \text{Subject to} \quad & 2x_1 + x_2 \leq 4 \\ & x_1 + 2x_2 \leq 4 \\ & x_1 - x_2 \leq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

2. Find the dual to the linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & x_1 + 2x_2 \\ \text{Subject to} \quad & x_1 + 2x_2 = 6 \\ & x_1 - x_2 \leq 3 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

3. Find the basic solutions of the following system:

$$\begin{aligned} \text{Maximize} \quad & x_1 + 2x_2 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 = 6 \\ & x_1 - x_2 + x_3 = 3 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

4. Let $A_{m\times n}$ be a matrix such that rank $(A) = m$. Consider the linear programming problem:

$$\begin{aligned} \text{Minimize} \quad & c^T x \\ \text{Subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

   A basic feasible solution is degenerate if it has more than $n - m$ zeros.

   (a) If two different bases correspond to a single basic feasible solution then show that it the basic feasible solution is degenerate.

   (b) If a basic feasible solution is degenerate then does it correspond necessarily to two different bases ?

5. Let P be the primal linear program in cannonical form and D be its dual. Show that the dual of D is P.