Problems due: 1
Due Date: Friday October 17th, 2014.

1. Let $A_{2\times 2}$ primitive matrix. Prove the Perron-Frobenious theorem for $A_{2\times 2}$ by computing the eigen-values of A.

2. Let $A_{n\times n}$ be a non-negative matrix such that for each i, j there exists a $k \equiv k(i, j)$ such that i, j-th element of A^k satisfies
$$a^k_{ij} > 0.$$
   (a) Decide whether A is primitive or not.
   (b) Decide whether the conclusions of the Perron-Frobenius Theorem holds for A as in this question.

3. Consider n-vertices, say $\{1, 2, \ldots, n\}$ and a graph G_n on it. For each pair vertices i, j let $a_{ij} = 1$ if there is an edge between them and 0 otherwise. $A = [a_{ij}]$ is called the adjacency matrix.
   (a) Let $n = 3$ and let G_3 be the graph formed by : connecting 1 to 2 and 3; along with a connection from 2 to 3. Compute the adjacency matrix of G_3. Decide whether A is primitive and if so find the maximal eigenvalue given by Perron-Frobenius Theorem.
   (b) Let B be a matrix obtained from A by suitably normalising each column to have column sum 1. Let E be a matrix such that $e_{ij} = \frac{1}{n}$. The Google matrix (so called) is given by
$$G = dA + (1-d)E$$
   with $0 < d < 1$. The Perron-Frobenius eigen vector of G scaled so that the largest value is 10 is called page rank with damping factor d.
   Construct G for the graph G_3 obtained in part (a) with $d = \frac{1}{2}$. Determine the page rank.