Problems due: X, *where* $X \in$ *Uniform*$\{2, 4, 6\}$
Due Date: Friday September 26th, 2014.

1. For any two subspaces S and T of a vector space V, show that

$$\dim(S + T) = \dim(S) + \dim(S) - \dim(S \cap T).$$

2. For any two subspaces S and T of a vector space V, show that

$$S \cap T = \{0\} \iff S + T \text{ is direct}$$

3. Show that for any two matrices A and B

$$\operatorname{rank}(A) + \operatorname{rank}(B) \geq \operatorname{rank}(A + B)$$

. From this conclude that for a matrix $P_{n\times n}$,

$$\mathcal{C}(P) + \mathcal{C}(I - P) = \mathcal{C}(P) \oplus \mathcal{C}(I - P) \text{ iff } \operatorname{rank}(P) + \operatorname{rank}(I - P) = n.$$

4. Find the matrix P such that it is the orthogonal projection onto the column space of A when A is one of the following:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & -2 \\ -1 & -3 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & -2 \\ -2 & 1 & -3 \end{bmatrix}$$

5. Find $(A^{\perp})^{\perp}$ for any set A of vectors.

6. Let

$$W = \left\{ \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix} : \alpha \in \mathbb{R} \right\}.$$

Let S be a subspace of $\mathbb{R}^3$ which is a translate of W. Find the orthogonal projection of of $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ into W and into S.