

Problems due: 5

**Due Date: Friday September 5th, 2014.**

1. Find necessary and sufficient conditions when  $\langle x, y \rangle = \sum_{i=1}^n \alpha_i x_i y_i$  is an inner product on  $\mathbb{R}^n$ .

2. Let  $m, n \in \mathbb{N}$  and  $\mathcal{P}_n$  be the set of all polynomials of degree less than or equal to  $n - 1$  over  $\mathbb{R}$ . Let  $A = \{a_1, \dots, a_m\} \subset \mathbb{R}$ . Decide whether

$$\langle p, q \rangle = \sum_{i=1}^m p(a_i)q(a_i).$$

is an inner product on  $\mathcal{P}_n$ .

3. Let  $V$  be a vector space and  $d$  be a metric on it. Does  $d$  always arise from a norm ?

4. Show that

$$\frac{1}{\sqrt{n}} \|x\|_1 \leq \|x\|_2 \leq \|x\|_1,$$
$$\|x\|_\infty \leq \|x\|_2 \leq \|x\|_\infty$$

and in general for  $1 < p < q$

$$c(n, p, q) \|x\|_p \leq \|x\|_q \leq \|x\|_p$$

for some  $c(n, p, q)$ .

5. Let  $S_{1,p} = \{x \in \mathbb{R}^2 : \|x\|_p = 1\}$ . Sketch  $S_{1,p}$  for  $p = 1, 2, 3, \infty$