Problems due: 1,6
Due Date: Friday August 8th, 2014.

1. (GS Ex 32, page 19) Use Gaussian elimination to solve

$$(a)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & -4 \end{bmatrix} x = \begin{bmatrix} 6 \\ 11 \\ 3 \end{bmatrix} \qquad \text{and} \qquad (b)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & -4 \end{bmatrix} x = \begin{bmatrix} 7 \\ 10 \\ 3 \end{bmatrix}$$

2. Show that interchanging two rows can be effected by elementary row operations of the other two types.

3. (BR Ex 11, page 162) Let $B_{m\times n}$ be a matrix that is obtained from $A_{m\times n}$ via elementary row operations. Is the transforming matrix P unique ?

4. (BR Ex 2, page 172) Reduce each of the following to a matrix in reduced echelon form by elementary row operations:

$$(a)\begin{bmatrix} 2 & 1 & 0 & 0 & 1 \\ 3 & 0 & 3 & 0 & 2 \\ 5 & 7 & -9 & 2 & 5 \end{bmatrix}, \qquad (b)\begin{bmatrix} 0 & 2 & 4 & 3 & 0 \\ 0 & 5 & 10 & 7.5 & 0 \\ 0 & 1 & 2 & 1.5 & 4 \\ 0 & 2 & 4 & 3 & 2 \end{bmatrix}$$

   Further, obtain the rank, a basis for the row space (from the rows of the matrices), a basis for the column space (from the column space of the matrices), and a rank factorisation for each.

5. Let $B_{m\times n}$ be a matrix that is obtained by reducing the l-th column of $A_{m\times n}$ to e_k for some $1 \le l \le n$ and $1 \le k \le m$ via elementary row operations. Prove that $i_1, i_2, \ldots i_p$th rows of B are linearly independent if and only if the corresponding rows of A are linearly independent, provided k is included in $i_1, i_2, \ldots i_p$

6. Suppose it takes k steps to reduce $A_{m\times n}$ to its echelon form (with r non-null rows) $B_{m\times n}$ via elementary row operations. Let v be the vector (described in class) that kept track of the interchange of rows. Show that $\{A_{v_1*}, A_{v_2*}, \ldots A_{v_r*}\}$ form a basis for the row space of A. Space basis.