Due Date: January 28th, 2008

Problems to be turned in: 1(b), 5

1. Using Lagrangian multipliers, find the maxima and minima of the following fucntions subject to the specified constraints:
   (a) $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = xy$, subject to $x^2 + y^2 = 2a^2$.
   (b) $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x^2 + 2y - z^2$, subject to $2x - y = 0, x + z = 6$.
   (c) $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x + y$, subject to $(x^2 - y^2)^2 = x^2 + y^2$.
2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ given by
$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \frac{x_1^3}{x_1^2 + x_2^2},$$
   when $x \neq 0$ and $f(0) = 0$. Decide whether f is differentiable at 0. Decide whether the partial derivatives $\frac{\partial f}{\partial x_i}(0)$ exists for $i = 1, 2$.
3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ given by
$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \frac{x_1 x_2^3}{x_1^2 + x_2^2},$$
   when $x \neq 0$ and $f(0) = 0$. Show that f is continuously differentiable at 0 and the partial derivatives $\frac{\partial^2 f}{\partial \partial x_i}(0)$ exists for $i = 1, 2$ but are not equal.
4. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a differentiable function such that $Df(x) = 0$ for all $x \in \mathbb{R}^n$. Show that f is constant.
5. Let $f : \mathbb{R} \to \mathbb{R}$.be the function defined by $f(x) = x + x^2 \sin(\frac{1}{x^4})$ for $x \neq 0$ and $f(0) = 0$. Show that f is differentiable at 0 and $Df(0) = 1$ but f is not increasing in any open set around 0. What is the significance of this example ?
6. Let $S = \{x \in \mathbb{R}^3 : x_1x_2 + x_2x_3 + x_3x_1 = -1\}$ Decide whether for $x_0 \in \mathbb{R}^3$ there exists a $g : \mathbb{R}^2 \to \mathbb{R}$ such that
$$S \cap V = \{\begin{bmatrix} x_1 \\ x_2 \\ g(x) \end{bmatrix} : x \in U\},$$
   where V is open in $\mathbb{R}^3$, $x_0 \in V \cap S$ and U is open in $\mathbb{R}^2$