

NOT Due

1. Consider the 2×2 non-linear system given by

$$\begin{aligned}x_1 - x_2 - 3 &= 0 \\x_1^2 - 20x_1 - x_2 + 5 &= 0\end{aligned}$$

- (a) Identify $A(x), b(x)$ such that the above can be written as $A(x)x = b(x)$. Modify `demoSSub` to obtain a solution of the same. Use 10 iterations and find two initializing vectors that give you the two solutions.
- (b) Identify f such that the above can be written as $f(x) = 0$. Modify `demoNewtonSys` to obtain a solution of the same. Use 10 iterations and find two initializing vectors that give you the two solutions.

2. Consider the matrices

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}, A = \begin{bmatrix} 0.1 & 0.1 & 10^6 \\ 0.2 & -0.1 & 10^6 \\ 0.1 & 0.2 & 0 \end{bmatrix}, b = \begin{bmatrix} 0.2 + 10^6 \\ 0.1 + 10^6 \\ 0.3 \end{bmatrix}$$

- (a) Using OCTAVE and the ∞ norm, compute (exactly) $\kappa(A)$. Decide whether the matrix A is ill-conditioned or not.
- (b) Using the `\` operator in OCTAVE, solve $Ax = b$.
- (c) Perturb a_{13} to get another matrix $A + \delta A$ such that $\frac{\|\delta A\|}{\|A\|} \sim 10^{-6}$. Using the `\` operator in OCTAVE, solve $(A + \delta A)\hat{x} = b$.
- (d) Compute $\frac{\|x - \hat{x}\|}{\|x\|}$
- (e) Let $\tilde{A} = AW$. Do parts (a) -(d) for \tilde{A} . Conclude that this matrix A was an “artificially” ill-conditioned matrix.