Due Date: February 21st, 2007
Problems to be turned in: 1, 2(b)-(f)

1. Starting with the code `GEshow` in the NMM toolbox develop a `GErectangular` function that performs Gaussian elimination *only* for a $m \times n$ matrix. The function should return $\tilde{A}$, the triangularised augmented matrix.

   Bonus Question[1] : Write a general program in OCTAVE, `GEgeneral`, which solves a linear $m \times n$ system of equations.

2. Centrifugal pumps are common devices used to move liquid through piping systems. The key question there is to determine the pressure head h of the pump given q the flow rate. Start with the model specified in `pumpcurve` code in the NMM toolbox.

   (a) Consider q and h from the following table:

q(m^3/s)	0.0001	0.00025	0.0008	0.001	0.0014
h(m)	115	114.2	110	105.5	92.5

   Using the first three data points, write down the equation you get between h and q.

   (b) Modify the `pumpcurve` function to accept q and h vectors of arbitrary length as input. Using all data points (above) except the fourth, use your function to find the cubic polynomial interpolant. Also find the condition number of A. Let c be the coefficents of the polynomial.

   (c) Replace the second point to 114 from 114.2. Do as in previous part to get $\tilde{c}$.

   (d) Find the relative difference vector $d = \frac{\tilde{c}-c}{c}$ for all i.

   (e) Plot h vs q, and find the largest difference in the value of h from 100 points between $\min(q)$ and $\max q$ in both the cases.

   (f) Discuss the practical significance on the pertubation of h values on the coefficient c and the values of h obtained by the interpolation function.

3. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & \epsilon \end{bmatrix}$.

   (a) Compute the condition number of A using the 1 norm or ∞ norm.

   (b) Deduce that the matrix is ill-conditioned for small ϵ.

   (c) Let $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\delta b = \begin{bmatrix} 0 \\ \epsilon \end{bmatrix}$ Show that

$$\frac{\| \delta x \|}{\| x \|} = \kappa(A) \frac{\| \delta b \|}{\| b \|}$$

[1]Eventual Prize: Carrot Cake at end of semester

At the end of this chapter you should be able to

1. Describe the most efficient procedures for solving $Lx = b$ or $Ux = b$ when L is lower triangular and U is upper triangular.
2. Name the solution algorithm most commonly used for solving $Ax = b$.
3. Write the equation that defines the residual vector.
4. Describe the significance of $\kappa(A)$ on the reliability of the numerical solution to $Ax = b$.
5. Describe the reason for pivoting. Is pivoting a remedy for ill-conditioned systems?
6. Estimate the number of correct significant digits in the numerical solution to $Ax = b$ given values of ϵ_m and $\kappa(A)$.
7.
8. State conditions required for a successful LU factorization of A. Write (describe) a procedure for solving $Ax = b$ given an LU factorization of A.
9. State conditions required for a successful Cholesky factorization of A. Write (describe) a procedure for solving $Ax = b$ given a Cholesky factorization of A.
10. Use OCTAVE and the LU factorization of A to solve several systems of equations that have the same A and a sequence of different b.
11. Use OCTAVE and a Cholesky factorization of A to solve several systems of equations that have the same A and a sequence of different b.
12. List the order of magnitude work estimates for Gaussian elimination with back substitution, LU factorization, and Cholesky factorization.

1. Using the code of the `lupiv` function in the NMM toolbox (directory –`linalg`) solve for x when $Ax = b$ when

$$A = \begin{bmatrix} 2 & 4 & -2 & -2 \\ 1 & 2 & 4 & -3 \\ -3 & -3 & 8 & -2 \\ -1 & 1 & 6 & -3 \end{bmatrix} \quad b = \begin{bmatrix} -4 \\ 5 \\ 7 \\ 7 \end{bmatrix}$$

   Provide the commands used by you in your answer.

2. Using code of the `cholesky` function in the NMM toolbox (directory –`linalg`) solve for x when $Ax = b$ when

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

   Provide the commands used by you in your answer.