All the programs mentioned in this worksheet are available in the `rootfind` directory of the NMM toolbox.

1. Using `fx3n` and `newton`, find the root of the equation $x - x^{\frac{1}{3}} - 2 = 0$ with an initial guess $x_0 = 3$, with x-tolerance and f-tolerance to be within 5×10^{-16}. Note down the number of iterations required for convergence and the value of the root.
2. Let $r = 1$ and $s = 0.25$. Using the inbuilt function `roots`, solve for h, where $h^3 - 3rh^2 + 4sr^3$.
3. Consider the following system of equations, $A_{2\times2}x_{2\times1} = b_{2\times1}$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

Find the solutions using "\" operator, obtained when the elements of A and b are perturbed as follows:($\delta = 5 \times 10^{-9}$)

   (a) $a_{21} = 2 + \delta$
   (b) $a_{22} = 1 + \delta$
   (c) $a_{11} = 2 + \delta$, $b_1 = 6 + \delta$
   (d) $a_{21} = 2 + \delta$, $b_2 = 6 + \delta$

Does the operator return the correct result when $\delta = 100*$`realmin` ?

Due: February 14th, 2008
Problems to be turned in: 1, 3

1. Write a `Bisection(a)` function which takes in a real number a and finds an approximation to $\sqrt[3]{a}$ to within 10^{-4} using the bisection algorithm. What is the result for $a = 25$, and $a = 8$?
2. Write a `parabola(x,y)` function to automatically set up and solve the system of equationsfor a parabola defined by $y = c_1x^2 + c_2x + c_3$. The function definition should be
`function c = parabola(x,y)`

The function should take two input vectors x and y, each of length three, that define three points through which the parabola passes. The function should return

   (a) a vector c of the three coefficients.
   (b) a plot of the parabola with the input points shown on the graph.

Test your answer with the following points:

   (a) (-2,-1), (0,1), (2,2)
   (b) (-2,-2), (-1,-2), (-1,2)

3. Write a function `Newtonsp` that will approximate to within 10^{-4}, the value of x_0 which is the point on the graph of $y = x^2$ that is closest to $(1, 0)$.

At the end of this week you should be able to

1. Explain the role of bracketing. Write a simple equation that expresses the condition for finding a root in a bracket interval.
2. Manually perform a few steps of the bisection method. Identify the one situation where bisection will return an incorrect value for x as a root.
3. Manually perform a few steps of the Newton's method and secant method
4. Identify situations that cause Newton's method to fail
5. Describe the possible expressions for convergence criteria. Specify convergence tolerance for any function so that excessive (unnecessary) iterations of a root-finder are not performed.
6. Describe the procedure used by `roots` to find the roots of a polynomial.
7. Qualitatively compare the convergence rates of bisection, secant and Newton's method

To perform basic root-finding with OCTAVE you will need to

1. Plot any $f(x)$ as a means of graphically identifying the location of roots.
2. Write an m-file that evaluates $y = f(x)$ for use with `bisect`, and `secant`.
3. Write an m-file that evaluates $f(x)$ and $f'(x)$ for use with the `newton` function
4. Find zeros of a function with the `bisect` and `newton`,
5. Find roots of polynomials with the `roots` command.