1. You are waiting at Jayram Das bus stop. Route no. 222A arrives in the form a Poisson process of rate one bus per hour, and arrivals of Route no. 226 bus form an independent Poisson process of rate seven buses per hour.

   (a) What is the probability that exactly three buses pass by in one hour?

   (b) What is the probability that exactly three 226 buses pass by while I am waiting for a 222A?

   (c) When the maintenance depot goes on strike half the buses break down before they reach Jayram Das bus stop. What, then, is the probability that you wait for 30 minutes without seeing a single bus?

2. Shyam wishes to cross from the Gopalan Mall to the median of the road, i.e cross one lane of slow-moving traffic. Suppose the number of vehicles that have passed by time t is Poisson process of rate 1, and suppose he takes 5 minutes to cross.

   (a) Assuming that the Shyam can foresee correctly the times at which vehicles will pass by, how long on average does it take to cross over safely?

   (b) How long on average does it take for Shyam to cross to the other side

      (i) when one must walk straight across (assuming that the Shyam will not cross if, at any time whilst crossing, a car would pass in either direction);

      (ii) when the median in the middle of the road makes it safe to stop half-way?

3. Let $\{X_t\}_{t\geq 0}$ be a right continuous process on $\{0,1\}$. Let Y_n be the jump-chain associated to X. Suppose the holding times S_n are independent random variables exponentially distributed with parameters λ_n given by

$$\lambda_n = \begin{cases} \mu & \text{if } Y_n = 1 \\ \lambda & \text{if } Y_n = 0 \end{cases}$$

   (a) Describe the generator matrix associated with $\{X_t\}_{t\geq 0}$

   (b) Suppose the initial distribution X_o is given by (π_0, π_1) show that

$$\begin{aligned} P(X_t = 1) &= \frac{\lambda}{\lambda+\mu} + (\pi_1 - \frac{\lambda}{\lambda+\mu})e^{-(\lambda+\mu)t} \\ P(X_t = 0) &= \frac{\mu}{\lambda+\mu} + (\pi_0 - \frac{\mu}{\lambda+\mu})e^{-(\lambda+\mu)t} \end{aligned}$$