Due date: October 17th, 2013

1. Let $f : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be C^1 functions. Show that the following

$$u_t + bu_x = f, t > 0, x \in R \; u(0, x) = g(x)$$

has a unique solution given by

$$u(t, x) = g(x - tb) + \int_0^t f(s, x + (s - t)b) ds$$

2. Using the method of characteristics solve :

$$\begin{array}{lll} (a) & xu_y - yu_x = u, x > 0, y > 0 \in \mathbb{R}, & u(x, 0) = g(x), x \in \mathbb{R} \\ (b) & u_x + xu_y = u, x > 1, y \in \mathbb{R}, & u(1, y) = h(y) \\ (c) & 2xtu_x + u_t = u, t > 0, x \in \mathbb{R} & u(0, x) = x \\ (d) & u_x + \frac{y}{2} u_y = u, x \in \mathbb{R}, y > e^x, & u(x, e^x) = 1 \end{array}$$

3. Find $u : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ such that

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x}, \; t > 0, x \in \mathbb{R}.$$

with initial value $u(0, x) = 0$ and $\frac{\partial u}{\partial t}(0, x) = 4 \cos(5x)$.

4. Suppose $\alpha, f : \mathbb{R} \to \mathbb{R}$ and $\beta : [0, \infty) \to \mathbb{R}$ are C^1 functions. Find $u : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ such that

$$\frac{\partial^2 u}{\partial^2 t} = \frac{\partial^2 u}{\partial^2 x} + \beta(t)\alpha(x), \; t > 0, x \in \mathbb{R}.$$

with initial value $u(0, x) = f(x)$ and $\frac{\partial u}{\partial t}(0, x) = g(x)$.