Problems due: 1
Due date: August 8th, 2013

1. Let $a, b \in \mathbb{R} \cup \infty, -\infty$ with $a < b$. Let $f : (a, b) \times \mathbb{R}$ be a continuous function and Lipschitz continuous in x uniformly over compact subsets of t[1]. Let $t_0 \in (a, b)$ and $a_0 \in \mathbb{R}$. Show that the initial value problem

$$\begin{aligned} \frac{dx}{dt}(t) &= f(t, x(t)) \quad t \in (a, b) \\ x(t_0) &= a_0 \end{aligned}$$

   Further show that the solution is continuously differentiable in (a, b).

2. Let $a, b \in \mathbb{R} \cup \{\infty, -\infty\}$ with $a < b$. Let $f : (a, b) \times \mathbb{R}$ be a continuous function and Lipschitz continuous in x uniformly over compact subsets of t. Suppose y, z are any two (distinct) solutions to

$$\frac{dx}{dt}(t) = f(t, x(t)) \quad t \in (a, b)$$

   then y and z cannot intersect in (a, b).

3. Let $a > 0, \lambda > 0, b \in \mathbb{R}$. Let x be a solution to

$$\frac{dx}{dt}(t) = -ax(t) + be^{-\lambda t} \quad t > 0$$

   Find the $\lim_{t\to\infty} x(t)$.

[1] i.e for any compact subset of (a, b) (say) M, $\exists K_M$ such that

$$| f(t, x) - f(t, y) | \leq K_M \mid x - y \mid$$

for all $x, y \in (a, b)$ and $t \in M$.