Problems due:3, 5
Due date: August 5th, 2010

1. Find $x : [0, \infty) \to \mathbb{R}$ the solution to the IVP

$$\frac{dx}{dt}(t) + \frac{2x(t)}{200+t} - \frac{9}{5}(1 - \cos(t)) = 0, \; t > 0, \text{ and } x(0) = 5.$$

2. Solve the following Bernoulli equation given by

$$\frac{dx}{dt}(t) - \frac{1}{3}x(t) = tx(t)^4, \; t > 0 \text{ and } x(0) = -2.$$

3. Find the general solution to the differential equation

$$\frac{dx}{dt}(t) = 9 - x(t)^2, t > 0$$

Find the particular solution with $x(0) = 0$. Is there a limit as $t \to \infty$?

4. Find the general solution the the differential equation

$$\frac{dx}{dt}(t) + kx(t) - e^{rt} = 0t > 0.$$

5. Luthorium, Lu 495 , decays with a certain half?life t_L to an isotope element Kriptonite, Kr 491 , by emitting an alpha particle. Kriptonite?491 is also radioactive, and decays with a different half?life, t_K , into lead (Pb) and another element.

   We will start at t = 0 with one mole (say) of Lu, no Kr, and no Pb, and watch the system evolve. We want to know the maximum amount of Kr495 which is present at any one instant. Help him out. Write $x(t)$, $y(t)$, and $z(t)$, for the number of moles of Lu, Kr, and Pb, in the system at time t.

   (a) Based on the model and what you know about exponential decay, sketch graphs of x, y, z, as functions of t. (Suppose that the two half?lives are of the same order of magnitude.) What are the limiting values as $t \to \infty$?
   (b) Write down the differential equations controlling x, y, and z.
   (c) Solve these equations, successively.
   (d) Find the time at which the amount of Kr is maximal.

6. *Extra credit* The solution to $y' = y$ with $y(0) = 1$ is unique and given by $y = e^x$. What is the Euler approximation for e = y(1), using 1000 equal steps?